TA BL tî D ES SIA T I O N

UNIVERSITÉ PARIS 7 - DENIS DIDEROT

LES MODELES ASTRONOMIQUES

DE

CLAUDE PTOLEMY AND THEON OF SMYRNA

Master's thesis
Epistemology, History of Science and Technology
presented by

Sybil de MONTAGU

S

supervised by

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PRE FAC E

, located at ét u de , is on the translations of the following works:

- *μαθηματικη συνταξις* **de Claude Ptolémée**.

We will refer to them under the most commonly used title, namely *l 'Almaɛ‹•xi‹•".* We will use the translation by G . J. Toomer, based on Greek manuscripts and published under the title */'/o/ in i' '.› //ti‹/yc'.›'/** (see bibliography).

– Ine Ini are dec H ypotlt èscv d e Cla u d e Pt o1ém ée .

For the first treatise , we have based ourselves on the translation by Régis Morel based on the Arabic version published in *k1I13I,f1* (21 (1993), pp.7-85) For the second treatise , we used an unpublished translation by Régis Morel based on the Arabic version.

We have also used partial translations by BR. Goldstein at from the version g rec que published in *i'r‹m.soc iivti.v uf the Amvric'aii* *l*liilo.sopliic:‹y .$‘oc i‹•i* (N S 57/4 (1 9G7), pp 3-55) under the title "Fixe .Arabie \’ersion of Ptoleniv"s Planetary l hypothèses"

-Των κατα το μαθηματικον χρησιμων εις την πλατωνος

by T li éo ri de Sin y rn e.

We use deux tractions de c and o un'ray e, I unceffectée par J D puis et p nb l iec e n 1.8.9.2 sous l e t it re / ’xy›‹ .›iii‹›ii c/‹•.s *r•‹›iiii‹ii.s.s‹iiir•«.s muîh‹•”tyi‹ui‹ju«.* iHi/c’.s *|›r›iii” /‹-/ Ic•c'liirc•* ‹/c’ *I’l‹iI‹›ii’,* I”autre elTectuee par J o e II e D elat t re- B ienc ou rt d an s le cad re d*un e t h è sc d c d o ct o r a t en Le tt re s et Scienc es h u m*a*î n es. so ut en u e en 1 .99.7, ' *!l’hêou Jeu Sm,'ri«•, 1’ltiln.so the ›hiu›iii‹’ivii.* A/rN(•/r.› *mécuiiique.v eu u .str‹›iinnhe grec‹pic•* Both translations were made from the Greek.

For reasons of simplicity, we will refer to this category as Théon de S m Y rn e under the contracted title *'I.'x pri.s ition*

In the present study, certain references are listed at the bottom of the page. In addition to the page, the lines to which the references refer are listed at In the case of the *first* reduction of *Almuge.vie* par Too mer ces 1i3 nes so rii compt ees a part i r des po rt io ri s de l ed it io n du t ex t p rec p ar H eibe rg a miguel Too nier renvoie fr équem ni ent en mar de de sa t ra du ct i o ri N os ré t'è ren ces à des 1ig nes pour l a t ra du ct ion de lThéon by J Delattre are also listed in a similar manner. i.e., the lines are numbered based on markers referring to the Greek text placed in the margins of his translation by J Delattre Par c cint re, p nu r in trad u ct io rt de J Du p ui s. Ies I iyn are not rotated apart Î r c la t ra d ti cl i o ri fran ça i se .

In general, we expressed the values in the mixed sex-appeal system used by GJ Too mer the entire angle is expressed in the decimal system and indicated by a point followed by a decimal point ";". while the fractional part is expressed in the sexagesimal system, and the different fractions are separated from each other by virgules.

We have also used the usual symbols for to represent the zodiacal constellations. That is to say :

♈ : Aries
♉ : Taurus
♊ :
♋ : ~~poissons~~ cancer
♌ : Vierrue
♍ : bal an ce
♎ : sccrpi o
♏ : n sa gil M
♐ : re
♑ : capri
♒ : Aquarius
♓ :

INTRODUCTION

The importance of Claude Ptolemy's work can hardly be overestimated. The Alexandrian astronomer's writings represent both the pinnacle of Hellenistic astronomy and the foundation upon which Western astronomy would develop over the next millennium and a half, first in the Arab world and then in Christian Europe. It is the first aspect of his importance that we will focus on in this paper.

Rolémée's work follows in the *tradition* of works that can be said to have originated with Plato (429-347 BC). Indeed, according to Simplicius, it was Plato who asserted that the stars must move only in circular and uniform motions, due to their divine nature. The first astronomer to attempt to explain planetary movements on the basis of this postulate was Plato's contemporary Eudoxus of Cnidus (lived 408-355 *BC,* active around 360-355 *BC). He* assumed that each sphere was *fixed at* the equator of a sphere whose movement was coupled with that of several other spheres. All were centered on the Earth but had different axes of rotation and turned in opposite directions. Eudoxus thus arrived at a universe with a total of 27 spheres. His disciple Callippus of Cyzicus (lived around 370-300 BC) modified this system by adding several other spheres—two for each element [4] and one for each of the *five* planets—in order to obtain a better correspondence.

1 In his *Commentary on Aristotle's Physics*, Simplicius writes: "Plato [...] poses this problem to mathematicians: What are the uniform and perfectly regular circular motions that should be taken as hypotheses, so that we can save the appearances presented by the wandering stars?" Jean-Pierre Verdet, £/ne JI*JfOfre *de l'astronomie, p.*41.
2 Thomas Hcath, *Ari,s4archus ofsaiiios, the Anciens C'operiti‹us,* p.191.
3 Thomas Hcath, *Arisiare'hus ofsamos,* p.212.
4 the sun and the moon.

with the movements observed. *He thus* obtained *a* system of 34 spheres. Aristotle (who lived around 384-321 BC) further complicated it to obtain 56 spheres—more than double Eudoxus's number—but for other reasons: his primary concern was to respect the principle, also established by Plato, that the movements of the planets are independent of each other, whereas, since the spheres touch each other—because Aristotle considered the possibility of a vacuum absurd—an outer sphere necessarily communicates its movement to the sphere within it. This led him to introduce numerous 'compensating' spheres to cancel out, at the level *of* each star's system, the effect produced by the movements of the spheres *above it.* Greek astronomy then underwent a major change with the development of two 'hypotheses': that of the eccentric and that of the epicycle. As we will see in more detail in Chapter 3, both are expressed in reference to circles rather than spheres, which gives them a more abstract, more strictly geometric character. The first represents the movement of a celestial body by a circle whose center is no longer the Earth, and therefore no longer the center of the universe—an eccentric circle. The second uses two circles, with the movement of the celestial body resulting from their movements. The larger circle has the Earth as its center and defines the movement of the center of the smaller circle. The celestial body moves along this second circle. These two hypotheses were already known to Apollonius of Perga (second half of the 3rd century BC), — who demonstrated their equivalence. Hipparchus (active around 150-125 BC)([6] then used them to account for the movements of the sun, determining the parameters to be assigned to them. Hipparchus also developed a geometric model for the moon based on these hypotheses,([7]

More than two and a half centuries separate Ptolemy from Hipparchus, and yet little is known about the attempts that were made to account for planetary movements before the Alexandrian devoted himself to this task. It seems clear that the models proposed

5 Probably from around 240 BC until the end of the 3rd century. Indeed, according to Eutocius, he was active during the reign *of* Ptolemy Euergetes (246-221) (Eutocius, *Conmtenlury or Apollonius Conirs,* Apoll. Perg. cd. Hciberg ii. l f›8. 5-170. 2b), and it is likely that it is to him that Ptolcmaeus Chennus refers as having lived during the time of Ptolemy Philator (221-204) (apud Pholii *Bilil.,* cod. cxc.. cd. Bekkcr 151 b 1 8); in *Selections illustrating the History of Greek and Roman Mathematics,* Vol. II, pp. 27tî-277

6 Verdet, Cône *fiis/uire de l'asirnnnmie,* p.48.

7 Verdet, Chie *histriire de l 'astronomie,* ch. 5 (pp. 39-70).

by Rolémée were not surpassed in Antiquity, if only because they were still in use in the 4th century, as evidenced by the *Commentaries* of Theon of Alexandria on several books *of£ ' Almagest.* But what his contributions consisted of in his time is not clear. In this regard, we have three types of sources.

First, Indian astronomy is based on a tradition that predates Rolémée but postdates Hipparchus. It can therefore give us an idea of the models that were developed in the interval between the work of these two astronomers. In particular, it suggests that astronomical tables had already been developed before those of Ptolemy.

Secondly, and this is our main source, Rolémée himself tells us about the work of his predecessors. He takes care to distinguish their contributions from his own. In particular, he refers to Hipparchus, for whom he clearly had deep respect. We thus know that the model of the sun and the discovery of the precession of the equinoxes are due to Hipparchus. Rolémée also allows us to appreciate the model of the moon adopted by his famous predecessor, and his own contributions in relation to the latter. The fact that he breaks with the model of the moon developed by Hipparchus suggests that this model was still in use in his time. On the other hand, Rolémée considers the existing models for the five planets to be too unsatisfactory to provide us with any information about them.

We must therefore turn to other authors for information on the subject. To this end, the work of Theon of Smyrna, Tov xmœ zo pn8qptincov /prjœp« v ciç v zWu voç cv cxrtv, is of great interest. Indeed, it was most certainly written shortly before Rolémée's work. According to Toomer, *Z'Almageste* — or *Syntaxe Mathématique, Traité Mathématique,* po8tjttnTt xT} otivïaf;tç from its original name — Ptolemy's major work on mathematical astronomy, was composed in the 150s His

8 G. J. Toomer, *Ptolyiny's Almogest,* p.420, note 6. In *ounces,* as indicated by Tr›omer, PioJéméc *taii* iui -rriëmc refers to such tables.
9 Translated by *1.* Dupuis 'Em *i.ciii* ri *des connaissances mathématique.x* u//de.s *pour la lecture de Platon'* and by Joüllc Dclattrc Or *ce qui Vsi utile du yoint de vue .ccientifique à la ferrure de Platon'.*
10 Toomer, p.1. The reason for this is that the Canobian inscription, dedicated to Rolémèe, appears to predate *the Almagest,* whereas it dates from 146-147. We also know that *the Almagest* was written under the Emperor Antoninus (138-161).

Other astronomical works, such as the *Zívrr de.s Iłypolłièsc•.s, the Anulemina,* the *I°lunisyhaerium, the Tétrahiblv,* and the *Table.s |ùciles,* are later *works*. There is greater uncertainty about the period of reduction of Théoii's work. Dupuis believes that Théon must have lived at the beginning of the second century AD. (2)

It is true that, due to its non-technical nature, Théon's presentation is not as detailed as one might wish. It is more akin to *what* we would call today a popular science book. Furthermore, as its title indicates, its *specific* aim is to provide the elements necessary for a reader to understand certain passages in Plato's work, which was written nearly five centuries before Theon's time. it is clear that the latter *was* not concerned with *providing a* more detailed *explanation*.

Theories of astrology. We will be more concise on the aspects that interest us most: models that attempt to account for the movements of five planets. We can nevertheless infer the general aspects of these models, even if we cannot be certain that more specific aspects are accurate.

interest us most: models aimed at accounting for the movements of five planets. Nevertheless, *we* can infer the general aspects of these models, even if we cannot be certain that more specific aspects not mentioned by Theon had not been proposed.

We will therefore begin by studying, in the first chapter, the conception that Theon and Ptolemy had of the universe in general: its shape, its movement, and the position of the Earth within this universe. We will also discuss a number of fundamental concepts that were essential reference points for the practice of astronomy. The second chapter focuses on the two hypotheses that form the basis of the geometric models used to explain the movements of the stars: the epicycle hypothesis and the eccentric hypothesis, in their simplest forms. We will thus see how Theon and Ptolemy demonstrate that they can account for the irregular movements of the planets and luminaries in terms of uniform circular motions—and do so satisfactorily for the sun—and how astronomers demonstrate their equivalence. In a third chapter, we will describe the geometric models

11 Ils font en effet référence à *L'Almageste*, ou apporte des modifications aux résultats obtenus dans ce dernier.
12 Dupuis, p.v. Il se base pour cela sur le fait qu'il fait référence à Trasylle et pas à Ptolémée, qu'il devait donc être postérieur au premier et antérieur au second.

proposed by Theon and Rolémée to account for the movements of planets and luminaries in longitude and latitude. The fourth chapter will deal with several phenomena of particular interest to Hellenistic astronomy: eclipses, stations and retrogradations of the five planets, the maximum elongations of Venus and Mercury, and finally, the risings and settings of stars and planets. The fifth and final chapter will focus on Theon and Ptolemy's physical conception of the universe based on the geometric models they adopted.

CHAPTER I: THE UNIVERSE OF PTOLEMY AND THEON OF SMYRNA

Claude Ptolemy and Theon of Smyrna shared the same views on the general l’ organization of the universe. Like most Western astronomers of their time, they conceived of it as limited by a rotating sphere to which the stars were fixed—the sphere of the fixed stars. The Earth itself was spherical and stood motionless at the center of this universe, relative to which it was tiny. The reasons why they adopted these ideas are the subject of this chapter. We will compare the arguments put forward by Ptolemy and Theon, trying to follow the order of their presentations, which itself reveals the importance relative importance they attributed to these questions.

I - The pericity of the universe

Ptolemy and Theon both begin by demonstrating the sphericity of the universe. Ptolemy's approach is, however, somewhat more dynamic than that of Theon. Ptolemy insists from the outset not only that the heavens are spherical, but also that they "move like a sphere." Although this expression may seem absurd in itself,[2] its meaning is clear: the heavens are animated by a rotational movement and, because of their spherical shape, this movement does not alter the distance of the stars from the center of the universe—i.e., the Earth. The fact that Ptolemy refers to the movement of the heavens at the same time as their shape is indicative of the link between the two problems, of which Ptolemy is aware:

I Th5on dc Smyrne, *Exhibition,* translation by 1. Dupuis. pp. 100 & 201, Ptolemy, *Alalugesia.* tfaductiun dc G. 1. Turn, pp.38-4fJ.
2 Ptnlêmûe, / fi//oyc.sue, translation by CE. 1. 1'‹1t›mcr, p.CB, HT, l. I4- Iñ.

It is from the way the stars move that we can conclude that the universe is spherical. The fact that Theon does not *use* the expression indicates that he is less aware of this, but it is just as true for him, as his argument shows.

Theon presents fewer arguments than Ptolemy, and one might think that he chose those that seemed most convincing to him. He offers three—which he attributes to Adrastus. All are astronomical in nature. Ptolemy also presents physical arguments, but only at the end. It is therefore likely that much less value was placed on physical arguments.

Both astronomers give the same argument first, the one that makes it intuitively obvious that the heavens are spherical: we see the stars rise and set at the same points on the horizon and move along the same trajectory.[4] Theon leaves it implicit that this is circular. In fact, this is more than just one argument among others for Ptolemy: its observational evidence leads him to consider it the reason why the very idea of a spherical universe was conceived. He thus gives it particular historical importance.[5] Of course, this is only true for stars, not for planets. The two astronomers *do not mention this at this* stage. This can *be seen* either as the fact that they are reasoning *on* the scale of a short period, a few days at most—after all, what is at issue is only the diurnal motion. and it is on this scale that it is observed—or, perhaps more likely, that this is an approximation on their part—since the movements of the ecliptic on the horizon are still limited. Perhaps they are also thinking mainly of the fixed stars, which represent the vast majority of celestial bodies. Moreover, Ptolemy, who develops the argument further, takes the trouble to emphasize that it is mainly the movement of the stars that leads to the conclusion that they are spherical.

? 1. Delattrc believes that Thcon cile Adraste.

4 Ptolomeo, *Almogeslv,* translated by G. J. Toonier, 1,3, p.38, H10, t.4-HI t, 1..S, Theon of Smyrna, *Eryt»'iiir›ii,* translated by J. Dupuis, p.199,1. J2-15, Theon of Smyrna, *Exposition,* translated by J. Dclattre. pp.111 - 112 {1 2tl, 1.13-14].

fi Ptolétnée,*f'Alrttcigesle,* translated by G. J. Tot›mer, 1.3, p.38, H10.1. J-5 & H l 1,1.6- 11.

of the universe. He also emphasizes the importance of the fact that a center of rotation can be observed—i.e., the pole. [6]

Neither astronomer considers the fact that other forms of the universe could just as easily give rise to the same phenomena: in their geocentric conception, these are solely due to rotation around the same fixed axis. A rotation certainly implies that each celestial body describes a circle which, in the case of fixed bodies, is always the same. But it does not imply that these circles are of such a size that they form a sphere. We will return to this problem when we discuss the last argument put forward by Theon.

11 It should also be mentioned that Theon seems to give this first argument a much broader meaning than Ptolemy: he presents it as demonstrating not only the sphericity of the heavens but also that of the Earth, the idea that the Earth is at the center of the universe and that its size is negligible in relation to the latter. He seems to consider this set of hypotheses as the only one compatible with the diurnal motion of the celestial bodies. It is difficult to see why this should be so: neither the sphericity of the Earth, nor its negligible size, nor its central position are necessary for the regularity of the trajectories of the stars and their circularity. It is *in* fact quite possible that Theon intended the argument to address one of the hypotheses (the sphericity of the heavens) rather than all of the hypotheses that make up his conception of the universe. ([7])

On the other hand, the second argument offered by Theon (and Adrastus) is indeed based on .on the necessity that the Earth be located at the center of the universe and that its size be negligible in relation to it—but still not on the fact that it is spherical. Theon notes that the visible part of the sky is always a whole. He understands this as an argument in favor of the sphericity of the universe: "novioç." Ptolemy, however, does not present this argument, and we can undoubtedly attribute this silence to

ó *Pioté nele, I 'Alittngest‹•,* translation by Ci. *J. Toonier, 1,d.* p.S8, H *IO,l l* 2- H. f f , L8.
7 Th nn du Smymc, *Wpo,siion,* traducfic›n de J. Dupuis. p.100, l. 10-12: Thñnn dc Sinyrne, *Wposiiion,* translation of 1. Delattru, g. l l 1 20, l. 10-12].
h Th0on dc Sinyrnc. *Fwfi›‹»liion,* translation by J. Dupuis, p. I III, l.11.

This means that it is not valid, contrary to the impression given by the idea of symmetry that it suggests. In fact, the only way to determine that half of the sky is visible is to observe that there are pairs of stars diametrically *opposite* each other: regardless of which one sets. the other rises—which is only true for stars located on the equator (Fig. 1.1. a). This would be impossible if the visible part of the sky were less than half (Fig. 1.1 b). On the other hand, this could perfectly well happen with stars fixed on an irregularly shaped surface , even if the lesser degree of symmetry makes the notion of 'half the sky' less obvious (Fig. 1 . I c). What matters here is the position of the observer — and therefore of the Earth — and not the shape of the heavens

Earth at the center of spherical heavens:

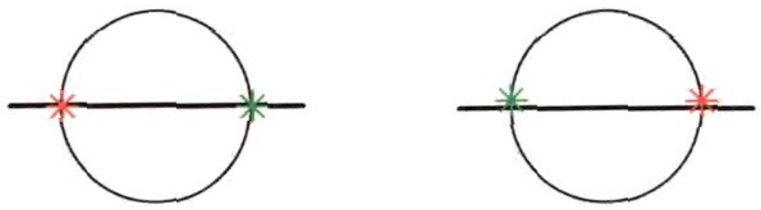

Fig. 1.1.a

Other positions of the earth.
Spherical heavens

Fig. 1.1.b

Earth at the center of irregularly
shaped heavens

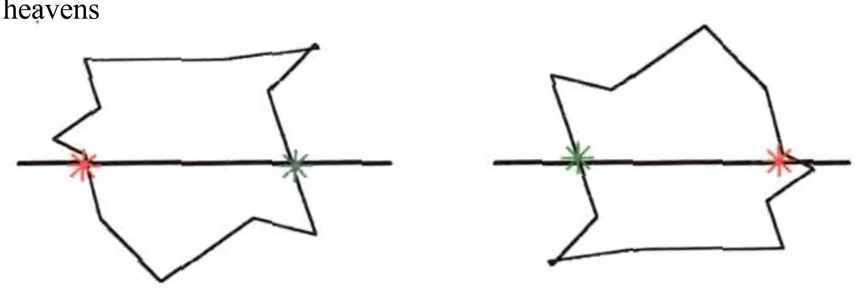

Fig. 1.1.c

However, Théon's last argument is also given by Rolémée. It is particularly interesting because it seeks to address directly the problem of the distribution of stars in the universe—in relation to the Earth, of course.[0] He is certainly inspired by the very definition of a sphere as having all points on its surface at equal distance from its center. The point is to show that stars always remain at the same distance from the Earth during their rotation. Theon affirms this, but it is not clear on what data he bases his assertion. Ptolemy, on the other hand, is quite explicit: the angular separations between pairs of stars, as well as their brightness, depend on distance. ([2]The fact that these two elements are constant for fixed stars regardless of their direction is proof for Ptolemy that the motion of fixed stars is spherical. And it is true that these observations clearly show that the distance of the fixed stars from the Earth does not vary—or at least not in a perceptible way. What Ptolemy does not indicate—which does not mean that he was not aware of it—is that this does not imply that they are all located at the same distance from the Earth. Theon, for his part, explicitly draws this conclusion: he asserts that a universe of any shape other than spherical would cause the distances of the stars from the Earth to vary as it rotates. This is not the case:

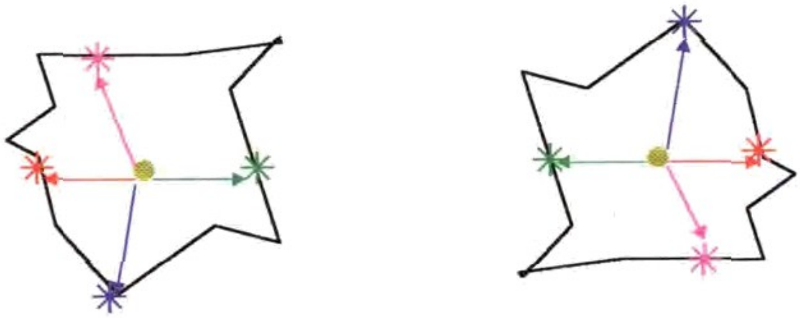

Fig. 1.2

9 Ptolémée, *l'Almageste*, traduction of G. J. Toooier, I, 3, p.39, Hl2, 116-H.13, 1.7;Théon de Surrne, *rpoñtion,* translation by J. Dupuis pp.199,1.19-21, 201 1.4-6: Theon of Sniyrne. Er ofioa, translation by J. Delatre., p.112, [120, 1.19-20 &120,26 27. ,121, 1.1].
Ptolemy will also use some of the same arguments to demonstrate that the earth is
‹) its negligible size in relation to the universe, given that it is located at the center of it. Due to its importance for œttn, this argument will also be used against
pessimism .
1111 refers at the same time to the argument concerning the visible half of the sky, and, once again, the reason for this is not clear.
12 Ptolémée, *l'Almageste*, traIntroduction by G. J. Tourner. I3), p. H.12. i 19 - H.13, 1.3.
13, 'if one assumes any motion whatsoever, except circular...', Ptolemy, f *Almapeste,* Waduction by G. J. Turn , I,3, p.39. Hl2,l.16.

Unlike Theon, Ptolemy does not assert that other shapes, necessarily less ethical than the sphere, would cause the distance of the stars from the earth to vary during their rotation. This silence, as well as the fact that he is aware of the distinction between the movement and form of the heavens—even if it is to affirm the link between the two—would tend to suggest that he may be aware of the weakness of this argument and of the two previous ones. This seems unlikely, however. Ptolemy certainly seems always aware of the flaw that the sphericity of the heavens is only a hypothesis whose foundations are not necessary, but he specifies '... absolutely all phenomena are in contradiction to the alternative notions which have been propounded.' His insistence ("absolutely all") suggests that he considers this absence of a valid alternative theory as proof of their non-existence.

As we have said, Ptolemy presents other arguments, which Theon does not mention. Some of these can be described as physical, and most of them are also aesthetic considerations.

The first is in fact only a brief allusion to the fact that the sphericity of the heavens is the only hypothesis that can explain the functioning of sundials. Ptolemy does not explain why this would be true.

Ptolemy then explains that the movements[16] of the stars must be as free as possible, no doubt because they must be uniform [7]— and that the sphere is the shape that best allows this — obviously because it has no angles, which Ptolemy does not specify. We might be tempted to consider this argument physical because it refers to the ease of movement of the stars, but it seems that Ptolemy himself does not consider it as such: he reserves the term for other arguments that follow, which refer to the

14 Ptolémñe, / *'Almageste,* translation by €i. J. Toomer, I. 3. p.38, H11, 1.9-1 1.
I •i Ptol6mcc, */'Almagest,* translated by G. J. Tooiner, I, 3, pp.59-40. H1 3, 1.d-H 14, 1.13.
16 Ptol6mce, *I'Almagesie,* translated by G. *1.* Toomer, 1, 3, p.39, H13, 1.9- l0.
~~17 This is all the more likely given~~ that Ptolemy concludes another argument as follows: '... it is plausible that the aether surrounding them [celestial bodies] [...] is spherical, and the majority *of the like Hfiis yarts moves in a circular and uniform fashion.'* (italics *added*). Ptolemy, *Almagest,* translation by G. 1. Toomer, I, 3, p,40, H14, 1.12-13.

composition of bodies. This is certainly significant in terms of his conception of the classification of sciences: it seems that for him, the study of motion was rightly considered part of geometry.

Ptolemy explains that among solids, the sphere is the one with the greatest volume for a given surface area. This does not present a physical advantage for the motion of the stars, but it seems to make the sphere the most appropriate shape for the universe, the largest volume that exists.

His final arguments are described by Ptolemy himself as 'physical'. They are all the more interesting in that they reveal his opinion on the composition of bodies. This was clearly atomistic. He explains that bodies other than ether are composed of parts that are 'round'—perhaps meaning spherical—but different from one another. The spherical parts that make up ether, on the other hand, are similar to each other and particularly fine. Ptolemy aims to deduce the form that ether must take on a large scale based on this characteristic attributed to its parts. By virtue of a principle that states that "bodies with parts like each other have surfaces with parts like each other,"(9) this similarity means that the parts of the surface of the body formed by the ether—i.e., the outermost part of the universe containing the fixed stars—must be as similar to each other as we would say today that this surface must be as symmetrical as possible.

possible. This naturally implies that it must be spherical. The spherical shape of the luminaries seems to confirm this conclusion: he believes that both the stars and the bodies that contain them are made of ether, and the spherical shape of the stars is confirmation for him that, on a large scale, ether takes on a spherical shape.

sphérique.

In addition to *these* additional arguments *in* favor of the sphericity of the heavens, Ptolemy also presents hypotheses against it, which he wishes to

18 Toomcr points out that this was proven by Zénodorus at the beginning of the 2nd century. G.J. Tooiner, *Ptolemy. Almagest,* p.4(I, note 25i. It is therefore unlikely that Ptolemy was familiar with this work.
19 Ptolemy, *Almagest,* translated by CL J. Toomer, I, 3, p.40, H14, t.2-3.

demonstrate the falsity. Theon makes no reference to these hypotheses. The first consists of the idea that the stars move from east to west in a straight line, either to infinity—which is incompatible with the fact that they reappear the next day—or in a soit back-and-forth motion—in which case we should observe this motion, which is also [20] incompatible with the fact that the stars appear closer 'bigger' and not 'smaller' when they are close to the horizon."

The other hypothesis criticized by Ptolémée wants que is that stars go out when they reach the horizon and are relit when they rise. As Toomer points out, if we are to believe Theon of Alexandria, this hypothesis seems to have been put forward by Heraclitus; Xenophanes was of this opinion, and the Epicureans considered it possible.[2] ' Ptolemy describes this hypothesis as absurd. He does not specify what is supposed to happen to the stars between their setting and rising. Perhaps they are supposed to be transported to an ocean surrounding the earth, an opinion that Heath already attributes to Thales.' Ptolemy's criticism is that the rising and setting of stars depend on the position of the observers: not only do the times at which they occur depend on this—being a function of the observer's longitude—but above all the very fact that they occur or not for given [24] stars—which depends on the position in latitude. This is, of course, totally incompatible with the purely objective nature of risings and settings according to this hypothesis, i.e., the fact that they are a state in which the stars themselves are found and not their relationship to the observer.

20 One might think **that a proponent of this** hypothesis could counter that the stars make this return ~~from west to east during the day~~ so that one cannot observe it, but this is not the case because we **are** trying to explain sunrises and sunsets **on** the horizon, and not the fact of greater or lesser brightness depending on whether the sun is up or not.

21 Here — actually a little further down, when he presents as an argument for the sphericity of the sky the fact that the brightness of the stars does not vary — Ptolemy explains this phenomenon by a kind of diffraction caused by moisture in the air — "the exhalations of moisture **surrounding** the earth" — which would have the same effect as water, which makes an **object** appear larger than it is when it is immersed in it. As Toomer notes (*Ptolemy's Almagest, p.* 39, note 24), Ptolemy expresses a different opinion in his *Optics (II,* 60), where he **attributes** the phenomenon to **a** purely psychological **cause**.

22 G.J. Toomer, *Ptolemy's Almagest,* p.39, note 23.

23 Thomas Heath, *Aristarchus of Samos. The Ancient Copernicans, y.19.*

24 Ptolêniëc **does not** specify on what observations he bases his assertion, but it **is**

certainement d'éclipse de lune.

ise a s hé de e

After discussing the shape of the universe, Theon and Ptolemy set out to prove that the Earth is spherical. For once, Theon's argument is more thorough than Ptolemy's. In particular, Theon clearly separates the problem of the spherical curvature of the land from that of the surface of the seas. In Ptolemy's work, the separation between the two problems is less clear, and he presents only one argument in favor of the sphericity of the surface of the seas. Furthermore, Theon responds to the objection that the Earth cannot be spherical because of the existence of valleys and mountains, which Ptolemy does not do.

However, when it comes to demonstrating the sphericity of the Earth in general (in Ptolemy's work) or from the perspective of land masses (in Theon's work), there are clear similarities between the two astronomers' arguments. Firstly, it is based on both astronomical arguments and physical reasons, but more on the former. With regard to the former, it is particularly interesting that both authors rely on the same facts, that the general line of their *argument* is quite similar, and that nevertheless their approach differs in the details. Indeed, Theon and Ptolemy both demonstrate the sphericity of the Earth by showing—with the help of astronomical observations—first that it has a curvature from east to west, and then that it has another curvature in the north-south direction.

Theon presents two arguments in favor of the convexity of the Earth from west to east. He first asserts that in eastern regions the same stars rise earlier than in those located further *west. He* also refers to observations of **lunar** eclipses**:** these are indicated as having occurred at

25 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.201, 1.8-10: Theon of Smyrna, **ExpositfOt2.** translation by J. Delattre, p.1 12 [121, 1.4-6].

different times, when in fact they occur at the same time. The further east observers are located, the earlier they report seeing the eclipse. [26] Ptolemy presents the same arguments in the same order, but clearly indicates that lunar eclipses constitute the observational basis for the first assertion, which is not clear in Theon.[27] Furthermore, Ptolemy emphasizes the implications of the fact that "... the differences in the hour are proportional to the distances between the places" [2] where the observations were made: he does not merely demonstrate that the Earth is convex from east to west, but also that this curvature is

regular [2]

Furthermore, Ptolemy reviews the other shapes that could be assumed, '
to show that they are inconsistent with the facts. Thus, he explains that if the Earth were curved but concave rather than connected, people in western regions would see the stars rise before those in eastern regions." If the earth were flat, all observers, regardless of their position, would see them rise and set simultaneously. ([1)] According to the sources we have, he seems to insist on this point, taking the trouble to cite several possible flat shapes:

> ...if it [the earth] were plane, they [the stars] would rise and set simultaneously for everyone on earth; if it were triangular or square or any other polygonal shape, by a similar argument, they would rise and set simultaneously for all those living on the same place surface."[(2)]

It is unusual for Ptolemy to explain an idea so thoroughly, and it is difficult to see why he would do so in such an obvious case as this, since his argument immediately appears valid for any flat surface. It is therefore likely that, in the original text, he was referring not to polygons, to different shapes of land

2ft Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.20i, 1. i o-1g. Theon of Smyrna, *fryoii/iori,* translation by J. Delattre, p.112 [121, 1.6- t l]
27 Ptolemy, *Almagest,* translation by G. J. Toonier, 1, 4, p.40, H.15, 1,2-11.
28 Ptolemy, *Almagest,* translation by G. J. Toomer, I, 4, p.40, H.13, 1,7-8
29 Ptolemy, *Almagest,* translation by G. J. Toomer, I, 4, p.40, H.15, 1,7-1 1.
30 Ptolemy, *Almagest,* translation by G. J. Toomer, I, 4, p.40, H.15, 1, l 3- 14 3
l Ptolèniée, *Almagest,* translation by G. J. Toomer, I, 4, p.40, H.15, 1.14- 15.
32 Ptolemy, *Almagest,* translation by G. J. Toomer, p. 41.

flat, but polyhedrons with triangular or square faces, suggesting that one could qu'imagine a cube-shaped, tetrahedral, dodecahedral, etc. Earth. This interpretation is reinforced by the fact that he refers to observers living on the *same flat surface,* although this could also be interpreted as 'on the same side, below or above the square, triangle'.

Ptolemy then considers the case of a cylindrical shape of the Earth, which, meeting the criterion of regular east-west curvature, effectively accounts for the observations he has mentioned so far. It is in this[33] context that Ptolemy introduces the notion of a convex curvature of the Earth in the north-south direction, whereas Theon addresses the question directly after demonstrating the convexity of the Earth from east to west, but does not consider different possible shapes of the Earth—which he did, however, for the universe, as we have seen. Theon's approach is therefore more of a deductive demonstration — i.e., since the Earth is convex both from east to west and from north to south, it is therefore spherical — whereas Ptolemy tends to proceed by refuting competing theories.[35]

L'Theon's argument in favor of the Earth's convexity in the north-south direction is based almost entirely on considerations of the circumpolarity of the stars. Implicitly placing himself from the point of view of an observer in the Northern Hemisphere, he explains that as one moves southward, more and more stars that were circumpolar around both the North Pole and the South Pole cease to be so. begin to set and rise: more and more circumpolar stars at the North Pole—which are always visible—become invisible for part of the night, while those that were circumpolar in the South become visible." Conversely, as one travels north, more and more stars become circumpolar: among those

S3 Ptolemy, *Almagest,* translated by G. J. Toonier. 1. 4, p.41, H.15, 1.18 - H 16. 1.10.
34 Theon of Smyrna, *Exposition,* translated by J. Dupuis, p.201, 1.19 - p.203, 1.8; Theon of Smyrna, *Wposiiion,* translated by 1. Dclattre, p.111| 12 i , I.12-28].
ñ4 We will return to this in relation to the two tronomes en particulier lorsqu'ils exposeront les **hyyothùses of the eccentric and** the eyicycle. See ch.2, I " partiu.
16 Thùon of Smyrna, *M/soz'ifion,* translation by J. Dupuis, p.OUI, 1.20 p.203, 1.2; Theon of Sinymc, ***Exf›osltion,* translated by** J. Delaltre, p. I I ñ 12 I , 1.1 ñ-22].

which rose in the south, many become permanently invisible, while in the north some of those that set become visible throughout the night. (7)

Although Theon does not say so explicitly, his arguments imply a regular variation. On the one hand, he refers to more and more stars. On the other hand, although he does not specify that it is the stars furthest from their respective poles that are the first to be affected, his explanation makes this so obvious that we can say that the idea is implicit. For the only concrete example he gives, which is that of Canopus, he specifies that "... it is always higher as one moves away from the North." **This** implies the more general argument that the maximum height above the horizon of the same stars varies regularly.

Ptolemy, addressing the problem of this north-south convexity in the context of the impossibility of a cylindrical Earth, begins by examining the consequences of an absence of curvature in this direction. He explains that for the inhabitants of the curved surface of the cylinder, there would be no circumpolar stars; all visible stars would rise and set.[40] Ptolemy makes reference to the changes observed when moving in a north-south direction, although more succinctly than Theon: he simply states that as one moves northward, more and more stars disappear in the south and more and more appear in the north.[4] He does not bother to specify that the opposite occurs when moving south, which he clearly considers self-evident, and concludes that the Earth curves evenly in the north-south direction.

north-south direction. 4[2]

The argument of Theon and Rolémée nevertheless has a weakness: their lack of data. It is *clear that* Theon and Ptolemy only have data for

17 Theon of Smyrna, *Exposition* translation by *1.* Duyuis, y.203, l,2-R, Theon of Sinyme, *E position,* Translation by J. Delaltre, p.113 [121, 1.22-28].

38 Theon of Smyrna, *Exposition,* translation by *1.* Dupuis, p.203, 1.7-8; Theon of Smyrna, *Exposition,* translated by J. Delattre, p.113 [121, 1. 26-28].

?9 Thion ***öe Smyroe,*** Myorzzi æ, translation by J. DupuJs, **p.2G?,** J. -2.

40 Plolémée, / ***'Almagvste,*** translation by G. J. Toomer, 1, **4, p.41,** H l G, 1.2-fi.

41 Ptolemy, *Almagvste,* translated by G. J. Toomer, 1, 4, p.41, H 16, 1.f›-7.

42 Piolëmëc, / *'A Image:ue,* untranslated **by G. J. Toomer,** 1, 4, **p,4l.** H 16, 1.7- 10,

better than a quarter of the Earth's surface as they can estimate based on this same assumption—which Ptolemy acknowledges elsewhere. "To assume, as they do, that the curvatures they observe form a complete circle is an extrapolation. Furthermore, while observations of circumpolarity of stars are sufficiently easy for them to be able to assert the regularity of the north-south curvature at the latitudes they know, the same certainly cannot be said of the east-west curvature: indeed, it is doubtful that Theon and Ptolemy had sufficient observations of lunar eclipses to assert their regularity, even in the part of the world known to them. In fact, nothing in the data available to them indicates that the Earth cannot have an irregular shape, concave in places. This is all the more true since, if their hypothesis of a spherical Earth is false, there is nothing to allow them to conclude that the Earth is not much larger than they think, and their extrapolation to other regions would then become much more hazardous. If they assume that the Earth is regular in shape, it is probably, on the one hand, because of the regularity that the considerations of the circumpolarity of the stars actually highlight. This may be why Theon insists so heavily on them. On the other hand, it also comes from an implicit principle of simplicity, which clearly involves the notion of symmetry.

The argument put forward by the two astronomers poses yet another problem. As Olaf Pedersen points out, " Ptolemy makes no reference to Aristotle's argument in his treatise on the heavens, namely that during a lunar eclipse, the shadow cast by the Earth on the Moon is always curved in a connected manner, whereas the phases of the Moon are also concave, i.e. crescent-shaped. ([4])The same remark applies to Theon. We are therefore entitled to wonder whether the two authors knew Aristotle's work directly or whether they were familiar with it only through his commentators or other Peripatetic authors.

43 Ptu]ëiT1ce, *1'Almageste,* translation by G. J. 'foomer, IJ, t, p.75, HS8, 1.1-4.
44 f Jlat' Pedersen, *A* .survep *ofthe Almagesi,* p.39.
45 Aristotle. *En the Heus•ens,* translated by W. K. C. Guthric,l I,14, 297b p.2.53.

In addition to these astronomical arguments, Theon also presents physical arguments. These are presented as two separate arguments, but they are in fact remarks that complement each other in such a way that one is tempted to assume that the two passages originally referred to the same argument. Theon starts from the idea that heavy bodies move towards the center—implicitly of the universe and, consequently, of the Earth. He specifies, in what is presented as the second "argument," that these bodies fall vertically "and converge towards the center of the Earth."[7] This is the entire content of this second 'argument', which, as we can see, cannot in itself lead to the conclusion that the earth is spherical. To do so, it must be established that each of these bodies ceases to fall at a distance from the center of the earth that allows them to distribute themselves evenly around each other in such a way as to form a sphere. Ceasing to fall when they reach the surface of the earth at their vertical point cannot guarantee this. ([4]) Theon therefore explains that very large bodies will be heavy enough to repel those around them; they thus raise what is around them while lowering themselves until they reach approximately the same level, and thus achieve equilibrium.

As we have said, Theon and Ptolemy take care to demonstrate that the surface of the waters is also curved and not flat, although the idea appears more incidental in Ptolemy. Thus, regardless of their extent and the area they represent in relation to the land—which neither of them specifies—they cannot detract from the spherical nature of the world.

Theon first relies on an argument based on several observations. He explains that a distant object seen from the other side of a liquid surface is less visible the lower one looks at it because of the curvature of the water.

4d What he **defines** as "forming equal angles with respect to the surface of the earth, on either side of the line along which they are composed"; Theon of Smyrna, *K posiiioit,* translation by J. DcJaitre. p.114
47 Theon of **Smyrna,** *Espasition,* translation by J. Dupuis, p.203, 1.22-27.
4d At least, not as long as Theon does not suppose that they fall from all sides, in a symmetrical manner.
49 Thcon dü SMyrne, ***Expo› iiion,*** translation by J. Oupuis, p.203. **1.11)-l9.**

"obstructs." Thus, something visible at eye level disappears, or is only partially visible, if one lowers oneself, and on a ship it is possible to see land from the top of the mast before it is visible from the deck.

The only argument that Rolémée presents in favor of the convexity of the water's surface is reminiscent of that of N, but it is not the same. Indeed, Ptolemy also takes the example of the shore seen from a ship approaching it. However, he does not address the problem from the point of view of the observer's height, but from that of the observed object. He reports that tall 'objects' such as mountains appear small and gradually grow larger, starting from their summit, 'as if rising from the sea itself in which they had previously been submerged...'. The two arguments can be represented as follows: Théon: :

Fig. 1.3.a

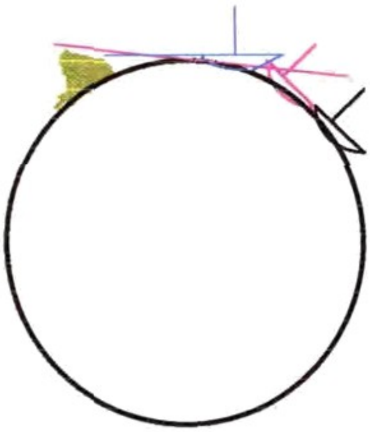

Fig. 1.3.b

qui une

Unlike Ptolemy sticks to this, **Theon** then offers[53] a demonstration that he describes as both mathematical and physical, which is reminiscent of the physical argument we have just seen. This new demonstration is based on the idea that water naturally flows towards the center

. Del tre. .2].
50 Thion & Saiyrne, Er con. tiadnction dc J. Dupui .205, 1.23. Théon de Smyrne. P.205. 15. Them de Smyme, fr hoa. .17 - . 1.7].
traductioii ‹k J st pp, HS [123, I .13-14.
51 Tbfon of Smyme, Erporifion, translation by J. Dopuis, p. 203, 1. 28 - p. 205, 1. 16-23. Théon de Smyrne, *Exposition*.
***W/›osffion,* translation by *I* Delattre, pp. 14-f. Il. [122, I** t24
52 Plnlëmée, f ñfmpgesre, translation& G. *1.* Toomcr. I, 4, p 41. Hl6. 1
53 Théon td Sioyme, *Exposition translation*&:J Dutri
translation ‹Je J. **£Kiattre,** pp 23 24

of the earth. However, unlike in the previous case, he does not take this for granted but takes the trouble to give reasons for his assertion, explaining that 'water always tends to flow from higher parts to lower parts. Now, the higher parts are further from the center of the earth, and the lower parts are closer to it.'"[4]Theon then uses a geometric argument from the absurd. He assumes that the surface of a body of water is flat and explains that in this case the points at the ends of the body of water are further from the center of the earth than the point in the middle. Consequently, the water will flow from the ends towards this point until it rises to the same height. Thus, all points on the surface will be at the same distance from the center of the earth, and therefore this surface will have a spherical curvature.[5] This argument is therefore similar to the one he puts forward in favor of the sphericity of expanses of land, due to the weighty nature of this element. The 'dynamic' nature of the reasoning should be particularly emphasized. Theon does not base his argument on a static principle of symmetry, where we would imagine a situation that is already in equilibrium, in which the homogeneous nature of bodies would allow us to conclude that they are distributed uniformly, symmetrically with respect to the center of the universe and the earth, thus forming a sphere. Although the reality that interests him is mainly static and in equilibrium, he feels compelled to resort to reasoning that starts from a situation of imbalance and to see where it should lead. Symmetry certainly comes into play, but as a tendency toward movement.
uniformly radial. This may be attributable to the fact that the physics it
knew made it difficult to conceive of a "force" as such, separate from the notion of motion.

Finally, as we have mentioned, Theon responds to the possible objection that the presence of valleys and mountains would make the notion of a flat earth untenable.

54 Theon of Smyrna, *fxyosifion,* translation by J. Dupuis, p.205. 1.8- 11.
55 Theon of Smyrna, *Exposition,* translation by *J.* Dupuis, p. 205. 1.6-23, Theon *of* Smyrna, *Exposition,* translation by J. Delattrc, p.115 [1 23, 1.4-124, 1.7].

spherical.'[6] His argument is that the depth of valleys and the height of mountains are too small in relation to the diameter of the Earth to influence its overall shape: they are merely negligible irregularities. To demonstrate this, he must therefore estimate the size of the highest mountains or deepest valleys and the diameter of the Earth. It is in this context that Theon addresses the problem of the size of the Earth. He takes the figure of 252,000 stadia calculated by Eratosthenes for its circumference.[7] without explaining the method used by the latter, and dividing it by a value of ri approximated at 3'/,,, he obtains approximately 80,182 stadia as the value of the Earth's diameter (instead of the 80,214 stadia that would be obtained for the same circumference with a modern value of ri). The data relating to the height of "the highest mountains above the lowest plains" are attributed to both Eratosthenes and Dîcéarque. According to Theon, they measured this height using a "diopter" and obtained a value of 10 stadia. Theon therefore concludes by comparing the two results that the height of the highest mountain is equal to approximately 1/8000 of the diameter of the Earth. Presenting the result in this way already implies that the relief of the Earth's surface has a negligible impact on the general shape of the Earth, but Theon does not stop there. He clearly wants to convince his readers of this negligible nature by giving them a mental image of this ratio. He therefore uses analogies. For example, he calculates that the ratio of "one fortieth of the diameter of a grain of millet" to a sphere one foot in diameter is equal to the ratio of the circumference of the Earth to the diameter of the Earth.

diameter is even greater than the ratio of the height of this highest mountain to the diameter of the earth," 0 then shows the negligible size of mountains in relation to the earth by comparing them in terms of volume, and concludes that 'a mountain 10 stadia high, which would be a sphere, would be much smaller in relation to the earth than

56 Thfon of Smyrna. *Exposition,* translation by J. Dupuis, p.205, 1.24 - p.211, 1.20; Theon of Smyrna, *Exposition,* translation by J. Dclattre, pp.1 l 6-1 20 [124, 1.7 - 127, 1.26)

57 This is the figure reported by Heron (Heron, *Oioptra 3b,* ed. H. Schöne 302, pp.10-17, in *Sélections illuzlraiing ihe Dis/ory of Green Mathematics,* Vol. II, p.273). Cleomcdes, for his part, speaks of 250tit30 stadia (Cleoinedes, *On the Circular Motion of the Heavenly Bodies i.* 10. 52, ed. Zieglet 94. 23-100, in *Selections if/usfra/in the History of Great Mathematics,* Vol. ff, p.273).

58 Theon of Smyrna, *Mposition,* translation by J. Dupuis, p.207. 1.22-25.

64,000 "part of a grain of millet, compared to a sphere one foot in diameter." Theon concludes by giving values for the size of the Earth: its circumference (252,000 stadia), its diameter (80,182 stadia), and its volume.

Ptolemy, for his part, is completely unaware of the problem of the impact of relief on the shape of the Earth. The reason for this silence is unclear. Perhaps he had not encountered the objection to which Theon undertakes to respond. Or perhaps he felt that he had implicitly answered it by referring to the changes in the circumpolarity of the stars depending on latitude, since if the irregularities of the Earth's surface (mountains, valleys) locally hide certain stars, the fact that observations allow us to speak of a regular curvature already implies that these irregularities are negligible.

III - The Earth is at the center of the universe

Having thus established the spherical shape of the Earth, Theon and Ptolemy set out to demonstrate its place in the universe: at its center. It is Ptolemy who deals with the subject most carefully. One gets the impression that Theon perhaps expected less skepticism on this subject from his readers than on the previous problem, as he deals with it very briefly. However, he presents two arguments in favor of the central position of the Earth. The first is astronomical in nature: wherever one is, half of the sky is visible "above" the Earth, which would not be the case if it were not located at the center of the heavens.[6] ' The argument is only valid if the size of the Earth is negligible compared to that of the heavens, but this is the problem addressed immediately afterwards, both by Ptolemy and by Theon. The latter offers a second 'argument': if the Earth were not at the center of the universe,

59 2'héofl of Smyrna, *ExflOS* fliOI1, translation by J. Dupuis, p.21 t , 1.2-6.
60 Theon expresses the volume of the earth with a precision that Ptolemy would disapprove of: 270,025,04? 508,297 and 1 l /21 cubic stadia. Theon also indicates the square and cube of the diameter of the earth, which is needed to calculate its volume. (Theon of Smyrna, *Exposition,* translation by J. Dupuis, p . 211, t . 14-20).
fi l Theon of Sinyrne. *Mpo.sition.* translation by J. Dupuis, p.21 1, 1.22-24.

'straight lines drawn from any point to the ends of the celestial sphere would not be equal'. 'Naturally, taken at face value, this proposition is not an argument but the very definition of what is to be demonstrated. We can in fact assume that Theon is implicitly referring to the same observations that led him to assert that the heavens are spherical, and that he did not give, whereas Rolémée explained that the constancy of the brightness of the stars and **their** respective angular distances made it possible to assert that their distance from the earth does not vary. Of course, as we have already noted, this does not imply that all stars are at the same distance from the Earth, contrary to what Theon asserts here.

Piolémée uses reductio ad absurdum reasoning in that he imagines the appearances that would result if the Earth were not located at the center of the universe. [6]Like Theon, he considers only astronomical arguments, neglecting those that Aristotelian physics might provide. He considers three scenarios, the third being a combination of the first two. He first considers the case where the Earth is located outside the axis of rotation (dium) of the universe while being equidistant from its poles, then the case where it is on the axis but closer to one pole than the other, and finally the case where it is neither on the axis nor equidistant from the poles.

pole than the other, [6] and finally the case where it would be neither on the axis nor equidistant from the poles·

By studying the consequences of a planet being outside the axis of rotation of the sphere of fixed stars, Ptolemy again distinguishes between two cases. For eela, he defined directions relative to an observer located on Earth: the Earth could be distant from the axis in the zenith-nadir direction, or in the east-west direction. Defining these directions as if they had an "absolute" meaning in the universe may seem surprising, especially since Ptolemy does not hesitate to insist elsewhere on

62 Theon of **Smyrna,** *Erposhion,* translation by J. **Oupuis,** p.211, 1.24-25.
63 Pioléciée, *l'lima et ie,* translation *by G. 1. Toomer, I, 5,* pp.4 -42.
64 Ptolemy, *Almagest,* translated by G. J. Toomer, 1, 5, pp. 41–42, H17, 1.7–H18, 1.9.
65 Ptolemy, *Almagest,* translated by G. J. Toomer, I, 5, p.42, H1 b, 1.10 - H l9, 1.1 3.
66 Ptolemy, *Almagest,* translated by G. J. Toomer, I, 5, p.42, H19, 1.14 - H19, 1.16.

the fact that directions such as 'up' and 'down' are merely linguistic conveniences that have no meaning when considering the universe as such, as we shall see below." In fact, this approach corresponds to his desire to show that phenomena, i.e. what an observer on Earth perceives, are different from what they are in reality.

Ptolemy therefore describes the observational consequences of a displacement of the Earth in the Nadir-Zenith direction, as seen both from the equator, i *sphaera* meta, and from other latitudes, at *sphciera rihliqiui.* The situations he envisages can be represented as follows.

A ***spliaera***
rectn:

Earth on the axis:

moved towards
the nadir:

moved towards the zenith

Fig. 1.4.a

A *spliaera obliqua:*

Earth on the axis:

displaced with the nadif:

moved towards the zenith:

Fig. 1.4.b

: axis of rotation of the sphere of
fixed points horizon
equator
. parallel bisecting the horizon

67 See below p.IS.
Gg Roldn ée. / '1/ i oçc•. ir. Translation by G. 1. Toomer. I. ñ. pp.41-42. HI7. 1.7 - H IB. l.4.

Indeed, if the Earth is located on the axis, half of the sky is always visible, whether for an observer located at the equator or at *.s' ›lnivr‹i nhliquo* the horizon divides the equator into two equal parts. In other cases of ti¿kure, this is not the case. If the Earth were moved towards the nadir, more than half of the celestial vault—and of the equator—would always be visible, whereas if it were moved towards the zenith, less than half would be visible, regardless of the time of day.

A *.vyha‹•rci rec'ta,* the horizon would not bisect either the equator or any of the parallels. However, it is along certain parallels—those sufficiently close to the equator, between the tropics—that the sun appears to make its daily rotations. A *.y›h‹iera reci‹i,* the sun would therefore never spend as much time "below" the horizon as it does "above" it, and there would never be an equinox (Fig. 1 5 Ia).

In *syhavrci ohliqun*, on the other hand (Fig. 1.5 lb), there could in some cases be equinoxes, if the parallels bisected by the horizon were close enough to the equator for the sun to move along them at certain times of the year—if the earth were not too far from the center of the universe. But this would of course occur when the luminary was at the points of the ecliptic that intersect the said parallels, and not the equator, as is normally the case. The equinoxes would therefore not occur in the middle of the period between the solstices, i.e., the moments when the sun is at its greatest declination from the equator—which would not be affected. Ptolemy explains that this is contrary to observations: the difference between the length of the day at the summer solstice and at the equinoxes is equal to the difference between the length of the day at the winter solstice and at the equinoxes, which indicates that the equinoxes occur exactly between the solstices.

G'2 If these were parallels located between the tropics

70 Roléniée specifies that the parallels in question are located either north or south of the equator, but does not provide any further details. In fact. From an observer located in the northern hemisphere, the first case corresponds to land located at the zenith, and conversely, the parallels would be south of the equator if the land were toward the nadir. The opposite would be true for an observer located in the southern hemisphere.

A *sphaera recta*:

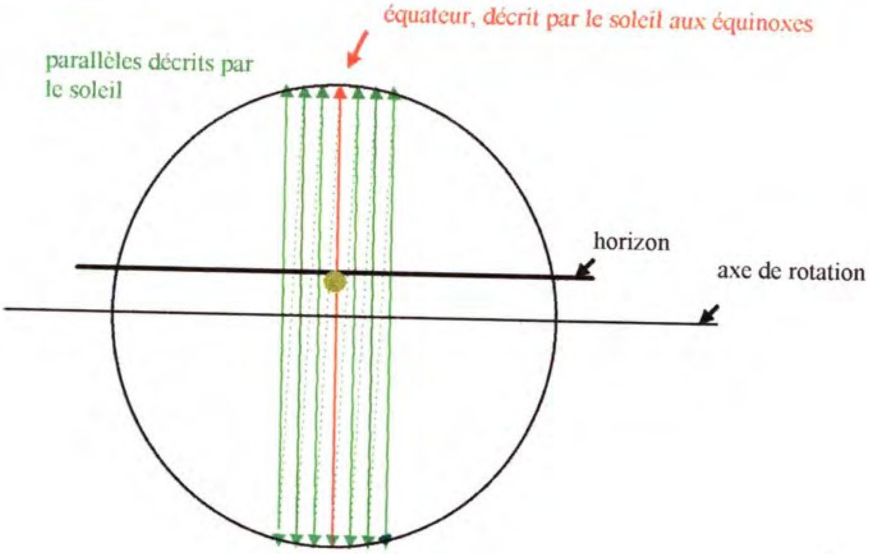

Fig.1.5.a

A *sphaera obliqua*:

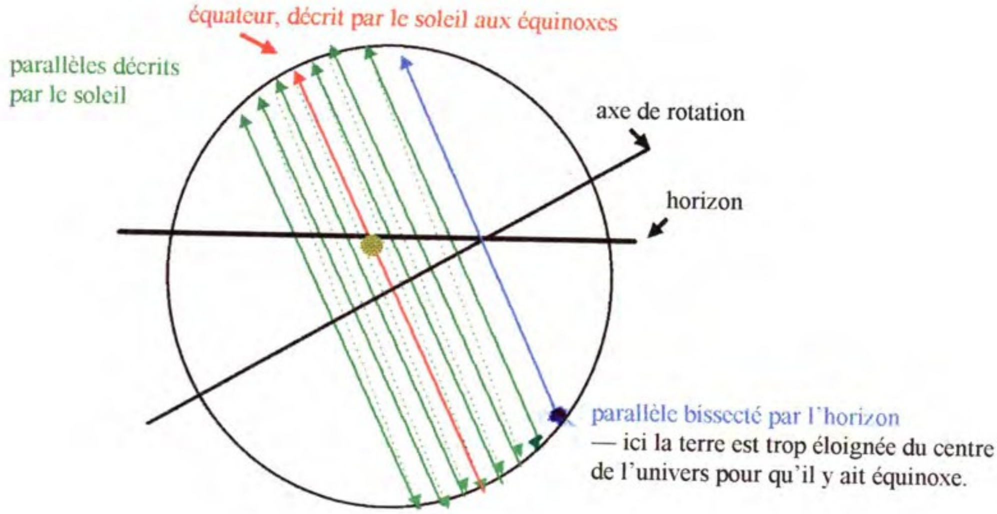

Fig.1.5.b

Ptolemy then considers what would be observed if the Earth were moved along an east-west direction [7] ' On the one hand. if the Earth were closer to the sphere of fixed stars on the eastern side than on the western side, the stars would appear to be larger and closer to each other when *they* rise than when they set (and vice versa if the Earth were closer to the sphere of fixed stars on the western side). This assumes, of course, that the Earth is significantly off-center, as a slight deviation would not be easily noticeable in this way. On the other hand, assuming that the Earth is closer to the east than to the west, the time between the rising and culmination of the stars would be shorter than the time between their culmination and setting.

Ptolemy does not mention the possibility of a combination of the two situations, i.e., a shift both in the north-south direction and in the east-west direction, but it goes without saying that the result would then be a combination of the effects described.

He then imagines the implications of the Earth's position on the axis of rotation of the sphere of fixed stars, but closer to one of its poles than to the other.

two problems. The first is again due to the fact that, in this case too, the part[72] of the sky above the horizon would generally not be equal to that below it. The ratio between the two would depend on the observer's latitude, as shown in the diagram below. Only for an observer located at the equator would the visible part of the sky correspond to the entire sphere of fixed stars—the horizon coinciding with the axis of rotation of the fixed stars (see Fig. 1.6). This would mean that, since the ecliptic is no longer bisected by the horizon, an observer located anywhere other than the equator would see either more or fewer than six zodiacal constellations; however, we always see six, regardless of where we are.

7 I Ptcléinéc, *L"Alniageste,* translation by G. 1. Toomcr, I, 5, p.42, HI 8, 1.4-5,
72 PtolëWe, *L* '.J/moges/a, translation by G. 1. Toomer. I, .'i, }t.42, HI 8, l.10 - HI 9, I.13.

axis of rotation of the sphere of fixed stars

: *horizons* at different terrestrial latitudes.

Fig. 1 6

A second consequence would be that at the equinoxes, the sun would not rise in the east and set in the west. Although Ptolemy does not specify this, it can be explained as follows. Consider the figures below (Fig. 1.7a and b)

Earth at the center of the universe.

équateur, décrit par le soleil aux équinoxes

SXC ÖC FOÏaÎiOfl

horixo

parallèles décrits par le

Fig. 1.7.a

Earth moved along the axis of rotation of the sphere of fixed stars:

Su

North

Fig 1.7 b

The equinoxes are defined by the fact that the sun then describes the equator in its diurnal motion. On other days, it describes other parallels—all located between the tropics. Therefore, at the equinoxes, the sun rises and sets at the points of intersection between the horizon and the equator. As Fig. 1.7.a shows, if the Earth is located in the middle of the universe, these points correspond to the terrestrial directions East and West. However, if it is closer to one pole than the other, this is no longer the case (Fig. 1.7.b).

Ptolemy concludes that, in addition to these considerations, if the Earth were not at the center of the universe, regardless of its position, eclipses could occur without the Sun and Moon being diametrically opposite in the sky, since the two bodies could be in opposition as seen from Earth without the line joining them passing through the center of the universe. It may be added that eclipses would not occur in the latter case, which is not clear from the passage in question in The Almagest. ([7] "

One might wonder why Ptolemy felt the need to demonstrate the central position of the Earth with such care, which he did not show when demonstrating its sphericity. It is conceivable that this was due to the nature of his audience, which was better informed about astronomy than the one addressed by Theon. Indeed, the argument that Ptolemy develops here serves as the basis for his argument on the immobility of the Earth—quite naturally, since the two problems are linked: it is difficult to imagine a moving Earth that is at the same time constantly at the center of the universe! However, whether or not the heliocentric hypothesis was popular among astronomers in the second century AD, it is likely that it was sufficiently well known to them to deserve more attention than Theon's readers required. It is therefore conceivable that this is why Ptolemy pays more attention to the problem of the central position of the Earth than Theon does.

71 Ptolémée, ***L'Almuyeste,*** translated *by* G. /. Tonn cr, f, S, p,42, H 10, I. î 7-H2fJ, I.2.

IV - The size of the earth is gligequal to that of the universe

It seems that the question of the size of the Earth relative to that of the universe was also unanimously agreed upon. Although Theon treats it less cavalierly than he does its centrality—he explains one of his arguments in more detail —he remains succinct. Furthermore, the way in which he expresses the negligible nature of the size of the Earth is similar to that used by Aristarchus of Samos:(74) "the volume of the Earth [has] no sensible *relation* to the extent of the universe, [...] it occupies only a *point* in this universe." The wording is similar in Rolémée: "...the earth has, to the senses, the ratio of a point to the distance of the sphere of the so-called fixed stars."

However, there is an important difference between Theon's idea and that of Ptolemy. In Theon, it is explicit that the size of the earth is negligible not only in relation to the sphere of the fixed stars but also in relation to / 'oz'fie *io/oirr.* [7] In other words, To use an expression that neither Theon nor Ptolemy use in the passages in question, for Theon there is no solar parallax. This is not the case for Ptolemy, who acknowledges its existence elsewhere. [78]Perhaps for this reason, Ptolemy presents the problem only in terms relative to the sphere of the fixed stars. However, he relies on the same argument as Theon, which does indeed concern the relationship between the size of the Earth and the solar orb: the points of the gnomons can be considered as the center of the Earth, regardless of where one is on it, so its radius is

74 As Archimedes tells us: '... the sphere of the fixed stars [...] has a size such that the ratio of the circle on which he supposes the Earth to revolve to the distance of the fixed stars is comparable *to the ratio of the center of the sphere* to *its surface.'* My italics, (Archimedes, f *'Areiiaire, text* established *here* by Charles Mugler, p.134).

75 Theon of Smyrna, *F,xfiO.S yiOH,* translation by J. Dupuis, p.211, 1.26-27. My italics

7fi Ptolcnióe, *L'Altttageste,* translation by *G.I.* Toomer, f, 6, p.43, H20, 1.4-5.

77 Theon of Smyrna, *Expus itiori,* translation by J. Dupuis, p.2 1 1, 1.26 - p.213, 1.3.

78 See ch.4, ı'' pattic.

79 Ptlemée, *L'Almuge.ele,* translation by G.I. Toomcr, *I,* 6, p.4.T, H20, 1.4-5

negligible.'[0] Ptolemy also extends this argument to 'amilliar spheres' — such as the *astrolabe* he describes later in / 'A/mageife'' — whose center can also be considered to be that of the earth. The explanation for this paradox certainly lies in the fact that although he attributes a parallax to the sun, it remains very weak.

Theon and Ptolemy both allude to the fact that we always see half of the sky — 'or nearly so', says Theon, the reason for this precision being unclear· [(2)]Theon presents this fact as incidental, as a consequence of the negligible size of the Earth, which he considers to have already been sufficiently demonstrated elsewhere. Ptolemy, on the contrary, uses it as an argument in its own right and takes the trouble to explain it further.

Ptolemy therefore repeats the same arguments as Theon, elaborating on them. However, the first argument he puts forward is not mentioned here by Theon, and Ptolemy seems to attach great importance to it: the brightness and angular distances between stars are constant.[4] As we have seen, this observation had already been used by *Ptolemy—and* Théon undoubtedly alludes to it—as an argument in favor of the sphericity of the heavens and the central position of the Earth. It is therefore hardly surprising to find it here: the argument amounts to saying that the surface of the sphere behaves like the center of the universe; despite the fact that, strictly speaking, two stars have a smaller angular separation on the horizon than when they are at the zenith, this difference is imperceptible. Given that this type of observation is now used to estimate the relative size of the Earth, Ptolemy specifies that angular separation and brightness are the same wherever they are observed on Earth: "it imposes which point on the Earth's surface behaves as the center of the Earth."

80 Ptolemy, *Aliftaye.ale,* translation by G. J, Toomet, I, 6, p.43, H20, 1.9- l4. 8 l
G.I Toomer, *Pt‹ilemy 's' Aliiiogesf, y.45,* rtote 36.
52 Theon of Smyrna. *Exposition,* translation by J. Ouptiis, p.213, 1.9-10.
83 Ptolemy, *The Almagest,* translated by G. J. Toomet, 1, 6, p.43, H20. 1.1 S-H2l , 1.S.
84 Rolémce, *L'Almoffeste,* translation by G. J. Toomer, 1, 6, p.43. H20, 1.5-9.
85 Ptolemy, *The Almagest,* translated by G. J. Toomer, I, 6, p.43, H2tl, 1.5-9.

V - **The immobility of the earth**

The way in which Ptolemy and Theon deal with the question of the immobility of the Earth is quite different this time. It is clear that Theon believes that the Earth is immobile. In fact, he notes that the Earth makes no sound because it is "immobile at the center." However, he does not bother to present any arguments against the idea that it could move. In the part of his work where he deals specifically with its central position in the universe and its size in relation to it, he does not even explicitly state that it is immobile. It is possible that he believes that by demonstrating that it is at the center of the heavens, he has thereby demonstrated that it is immobile. However, even if this *were* the case, there would be nothing to prevent him from stating it explicitly, which he does not do. Rather, one gets the impression that he does not bother to address this subject because he thinks that the idea of an immobile earth is not only familiar but obvious to his readers, for the reasons we have indicated.

Rolémée, on the contrary, does not fail to present arguments against the notion of a moving earth. The attention he pays to this subject in relation to Theon is undoubtedly due to a desire to provide a solid foundation for a notion that constitutes one of the pillars of the models he will subsequently develop with such care. In addition, this attention would also suggest that, while in the second century CE most educated people, to whom Theon was addressing, were either completely unaware of the heliocentric speculations of certain philosophers or were not sufficiently familiar with the arguments in question to feel the need for counterarguments, the same was not true of astronomers. However, as we shall see, Ptolemy does not seem to be thinking so much of Aristarchus's possible supporters.

86 Theon of Synnys, *Exposition,* translation *of 1.* Dupuis, p.25 *l, I. IS- ! h.*
87 Thcon of Smyrna, Expf9SAtion, translated by J. Dupuis, p.199. 1. t0 - p.213, t.IO,
h8 Ptolemy, *The Almagest,* **translated** by G. J. Toomer, I, 7. p.43-45.

0 first addresses the question of the movement of the earth through the heavens,[89] before studying that of the earth's rotation on its own axis." Regarding the former, his main *argument* is that since the Earth is located in the middle of the universe, it clearly does not move. This naturally rests on the idea that the arguments presented in favor of the Earth's central position are valid at all times, and thus the Earth *remains* at the center of the universe·

But Ptolemy also presents other arguments of a physical nature. These are aimed at people who find it unlikely that a body as heavy as the Earth could remain stationary without being supported by something, because they seem to be "making the mistake of judging on the basis of their own experience instead of taking into account the peculiar nature of the universe."[2]His counterargument is that this is not strange if we consider that, compared to the universe, the Earth is tiny, of negligible size, as he has already demonstrated. This argument is just as intuitive as that of his opponents: it seems reasonable that the earth 'should be overpowered and pressed equally from all directions to a position of equilibrium by that which is the greatest of all and uniform in nature'. This idea poses a problem of interpretation, because it is difficult to see what Ptolemy could be referring to here other than the ether. However, the ether is far from having a reputation for moving naturally towards the center of the universe, and it is therefore difficult to see how it could press the Earth from all directions. Ptolemy then explains that the impact on the earth of objects falling onto it cannot cause it to move, both because they are very small in relation to it and because they strike it from all sides. In both cases, he therefore uses the concept of equilibrium, and in a more 'static' way than Theon did, because the situations he is reasoning about are already in equilibrium and stable.

89 Ptolemy, *The Almagest,* translated by G. J. Toomer, 1, 7, pp. 43-44, H2l.d-H24, 1.1.
90 ***Ptolemy,*** *I '.Almagus1e,* translation by G. J. Tofimer, 1, 7, pp.44-45, H24, t.4-H26, 1.3.
9 l Ptolemy, ***L"Almagesle,*** **translation** by G. J. Toomer, I, 7, p.4ñ, H2l.8- 14.
92 Ptolemy, *L'Afinage.vie,* translated by G. *1.* Toomcr, I, 7, p.44, H22, 1.9-11.
93 Ptolemy, *Almapeste,* translated by G. J. Toomer, J, 7, p.44, H22, 1. 15-H21, f.l.

Ptolemy then defends the idea of the immobility of the Earth using reductio ad absurdum. He considers the case of an Earth that moves in the same way as "heavy" bodies, as the people he refers to intuitively understand, i.e., an Earth that falls. Starting from the Aristotelian—and intuitive—principle that the heavier a body is, including due to its size, the faster it falls, Ptolemy explains that the Earth would fall faster than any other object, and that other bodies would therefore appear to remain behind, suspended in midair. Ptolemy also explains that "the earth itself would very soon have fallen completely out of the heavens." One might already wonder, a s Toomer does, "what he means by 'out of the heavens': would it find itself outside the region of air, in that of ether, or outside the limits of the universe, outside the sphere of the fixed stars, if such a region were conceivable to Ptolemy and his contemporaries? In either case, Ptolemy ridicules the idea of talking about a body falling anywhere other than toward the center of the universe. He explains that the problem of the immobility of the earth is intimately linked to the way we conceive of falling and the directions 'up' and 'down.' His discussion on this subject is addressed to those, undoubtedly the same people, who would seek a cause for the movement of objects towards the center of the Earth, when it seems clear to him that this is due to the fact that the center of the Earth is the center of the universe. Ptolemy insists heavily on the fact that there is no such thing as up or down in the universe, but that these directions only have meaning for a given observer: the fact that all bodies fall down does not therefore imply that they all fall in the same direction relative to the universe; they all fall towards the center of the Earth.

94 Ptolemy, *The Almagest,* translated by G. J. Toomer, I, 7, p.4d, H23, 1 I 5- H24, 1.3.
95 Ptolemy, *The Almagest,* translated by G. J. Toomer, I, *p.44,* H2J, I,2. 9ô
G.J. Toomer, ***Ptolemy's Almagest,*** **p.44,** note 40.
97 Ptolemy, *The Almagest,* translated by G. J. Toomer, I, 7, p.44. H23, 1.1 -9.
98 Just like **Theon** regarding the sphericity of your earth, Ptolemy declares **that** bodies falling vertically, which he recognizes as forming right angles with the plane at the point of impact of an object — Theon defined it as "forming equal angles with respect to the surface of the earth, from each of the two **sides** of the line along which they are carried," which **amounts** to the same thing.

Ptolemy then goes on to demonstrate that the Earth does not rotate. [99]Not only are his arguments physical, but he even states that "there is perhaps nothing in the celestial phenomena which would count against that hypothesis, at least from simple considerations." Whatever less obvious considerations he may have had in mind, he does not present them. Some of his arguments were taken up in Europe in the 16th and 17th centuries in the debates surrounding Copernicanism. Piolémée argues that if the Earth were in rotation, any object not in contact with it—he gives the example of clouds or objects thrown or flying—would appear to move very quickly towards the West because they would be left behind. He offered a possible counterargument: for this not to be the case, not only would the air itself have to be in the same rotational motion as the Earth, but also any object in contact with the air would have to be welded t o i t . But if that were the case, objects could not move through the air, whether they were "flying" by themselves — such as clouds — or had *been thrown* (or rather, attempted to be thrown): they would appear motionless when viewed from the ground *because* they would not move through the air, which, being in rotation with the earth, would be motionless relative to it."

It should be noted that Pfolémée also has another type of physical argument: those related to the Aristotelian idea that the movement of an object depends on its nature.[0°] Thus, he considers it highly improbable that the 'rarest' and 'lightest' matter , i.e. ether , could be immobile while heavier and denser bodies are in motion. He also considers it improbable that a rarer and lighter matter , i.e. air could have the same motion as a heavier one—i.e., earth.'[(0)] 'Rolérnée does not fail to mention the facts that seem to contradict these ideas: he mentions that objects in the air—which he implicitly considers to be part of its nature—move faster than terrestrial objects. A

99 Piolinea,£ ***lmagest,*** translated by G. J. Toomer. 1, 7, pp. 44-45, H24, **1.4-H26, 1.3.**
10tl Ptolémée, *L'Almage,ste,* translated by G. J. Toomer, p.45.
O I Ptolbmée, *L* ***Almagesle,*** **translation by G. J.** Toomet, **I, 7, p.45,** H25, **1.6-H26,** 1.3.
102 Ptolemy, *The Almagest,* translation by G. J. Toomer, p.45,
103 Ptolemy. L'Almagesie, translation by G. J. Toomer, I. 7, p.45, H24, 1.1 fi-H25. 1.1.

What he is referring to is unclear. Rolémëe also notes that it is sometimes difficult to set certain terrestrial objects in motion, implying that it is even more unlikely that such matter could be naturally animated by rapid movement. The idea that it could have uniform motion also seems to bother him.[0] It seems clear that he finds these ideas highly ridiculous, and one gets the feeling that he is struggling to refrain from using the term. Nevertheless, he seems to consider the arguments relating to the movements of objects in the air relative to the earth to be more convincing than these Aristotelian considerations.

eVgloRe è o e

Although their arguments sometimes differ, we can see that Ptolemy and Theon share the same general conception of the universe: both believe that the Earth is spherical and immobile in the middle of the universe, which is limited by a spherical body made of ether, to which the stars are fixed. This sphere of fixed stars is endowed with a uniform diurnal rotation from east to west, which explains both the movement of the stars and that of the other celestial bodies it carries along with it.

Although they are undoubtedly less fundamental, it is nevertheless worth mentioning here a number of other general characteristics of the heavens on which the two astronomers also agree, as did not only their contemporaries but also many of their predecessors. Theon and Ptolemy define a number of other concepts that form the basis of their astronomy—to the point that we have already mentioned some of them. For example, they define the equator as "the greatest of the

t(l4 Ptolemy, L'A/inugezie, translation by G. J. Toomcr, I, 7, p.45, H25, 1.3-4.
105 Ptolémée. *The Almugeste,* translated by G. J. Toomvr, I, 7, F .45, H25, 1. 2.
106 or equinoctial, as Théon de SlTiyme, *Ktposition,* translation by J. DupUis, p.2 l5, l.S.

parallel circles described by the rotation of the points of the sphere of fixed stars.[07] Nights and days are everywhere of equal length when the sun in its annual course is brought onto this circle, which it then describes in the course of one revolution of the universe. [0] 'The regions of the earth for which the equator passes at the zenith always have nights and days of equal length.'"

In addition to this diurnal motion of the sphere of fixed stars, the two astronomers defined a second motion to account for the fact that planets and luminaries are not stationary relative to the stars, but move much more slowly from west to east, varying depending on the celestial body in question—being annual in the case of the sun. With this in mind, Ptolemy defined a large circle of the sphere of fixed stars, the ecliptic, inclined relative to the equator. For him, this circle strictly represented the

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course of the sun,"[0] whereas for Theon, as we shall see, "the path of the luminary is slightly inclined with respect to the ecliptic." Theon, moreover, does not mention the ecliptic as such, but defines the zodiac as "the band of the sphere of fixed stars within which the planets move." This amounts to the same thing, since he only needs to refer to the "middle circle of the signs" to refer to the ecliptic, the concept of which he clearly understands—although it remains strange that he does not provide a technical term for it. Ptolemy takes the trouble to explain why this zone must be defined—in his case, the ecliptic itself (5)—which Theon does not do. He explains that the movement of the planets from west to east does not

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107 **Ptolemy,** ***The Almagest,*** translated by G.J. Toomer, 1, 8, p.45, H26, 1.13-15; Theon of Smyrna, *Exposition,* translated by *I.* Dupuis, p.213, 1.23-27 & p.215, 1.6-10; Theon of Smyrna, *Exposition,* translated by J. Delattre, p.123, [129, 1.18 -24) & [1S0, 1.5-6).

108 Piolémée, *L'Almageste,* translated by G. J. Tourner, I, d, p.46, H26, 1.17-18, Theon of Smyrna, *Mposiiion,* translated by J. Dupuis, p.215, 1.10- **13, Theon of Sinyrne,** ***ExposiIf!On,*** translated by J. Delaitre, p.123 [130, 1..7- 10}

109 Theon of **Smyrna,** ***Mpositf!*****Ofl,** translation by J, Dppuis, p.215, 1.8- 10. Theon of Smyrnc, *Exposition,* translated by J. Delaltre, p. 123 [1 30, 1.ſ›-7].

110 Ptolemy,£'*Almagest,* translated by G. J. Toomer, p. 46, 1.10-11.

111 See ch.3, part 4.

1 12 **Theon of Smyrna,** ***Mposilion,*** translated by J. Dupuis, p.223, 1.8-9 (somewhat unclear); Theon of Smyrna, *MpOs Ation,* translated by J. Delattre, p.129, [1 35, 1. 14.lui).

113 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p. 215, 1. 20-30; Thëon of SlTiyrne, Exposfiion. translation by J. Delattre, p. 124, | 130, [.1 8 - 1 31. 1.3]

114 i.e. from the middle of the zodiac, which forms a band on either side of the ecliptic. 11 5

Piolérnëc, *L'Almageste,* translation by G. J. Toomcr, I, fi. p.4ſ›, H2ô, 1.15 - H29, 1.2.

does not occur parallel to the equator but varies the distance between the latter and the planets, which move away from or closer to it in a regular manner.[6] He further specifies that this deviation in declination cannot be explained[7] by the action of a uniform force. because the movement 'from that point of view is irregular' " which can certainly be interpreted as meaning that the speed at which the planets move away from or closer to the equator is not constant. Ptolemy thus justifies the hypothesis of a second movement of the sphere of fixed stars, around other poles than those of the equator.

Ptolemy and Theon use several other useful circles as reference points. Theon defines *the honzon* as the large circle that divides the sky into a visible part above the earth and an invisible part hidden by it.[9] Ptolemy does not bother to give a definition, but he notes, like Theon, that the horizon bisects the equator.[0] Theon also notes that it bisects the ecliptic, which Ptolemy does not specify but which is obvious since these are large circles.

The two astronomers also define the meridian as a large circle perpendicular to the horizon that passes through the poles of the equator and on which the sun is located at noon—and at midnight, as specified by Ptolemy.[22] Theon also calls this circle 'colum', whereas according to Eudoxus and Manetho, this term was reserved for two large circles.[2] One passes through the poles and the ecliptic and the equator, and

116 Ptolery,£ *'Almagest,* translation by G. J. Toomer, I, 8, p.46, H28, 1.5.
t 1 1 Jaiiludc measured relative to the equator.
1 1 8 Ptolemy, *The Almagest,* translated by G. J. Tourner, 1, 8, p.46, H2h, 1.7-d.
119 Theon of Smyrna, *frposilion, translation* by J. Dupuis, p.217, 1.1-5; Theon of Smyrna, *Myosiiion.* translation by J. Delattre, p.124 1131, 1.5] - p.l 25, [131, 1.9].
120 Ptolemy, *Almagest,* translation by CE. J. Toomer, I, d, p.46, H2ô, 1. J 5 - J f›. Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.217, I.5 -7; Theon of Smyrna, *Exposition,* translation by J. Delaitre, p. 125, [131, 1.9- l0J.
ï2 *Letter* of Smyrna, translation by J. Oupuis, p.217, 1.5 -7; Thcon of Smyrna, *Exposition.* translation by J. Dclattrc, p.125, [131, 1.9-10}.
122 Ptolêrnéc. L 'Afmogesir, translation by CE. J. Toomer, I, 8, p.47, H*9, 1.l7-H30, 1.1; Theon of Smyrna, *Exposition,* translation by J. Dupuîs, p . 217, 1.10-1f›: Theon of Smyrna, *Mposltion,* translation by J. Deiattr, p. iz5 [isi, 1 .14-1s2, l .s].
123 *TWon of* Smyrna, *E position,* translation by J. Dupuis, p.217, 1.15: Thëon of Smyrna, *Exposition.* translation by J. Delattrc, p.125, [132, 1.2).
124 G. *1.* Toomer, *Ptolvfny's* M?z rage.iz, p.19.

allows us to define the solstices: it cuts the ecliptic at these points, which are furthest from the equator. Theon simply refers to the tropics, circles parallel to the equator that pass through the solstices, which he defines only as "enclosing the zodiac."°(5) Ptolemy also defines the equinoxes, points of intersection of the equator and the ecliptic.°'

In addition to these differences in detail, there are also particular concerns specific to each of the two authors. Ptolemy demonstrates a desire to prove his point—particularly in his definition of the ecliptic—while Theon's approach is entirely descriptive. This is certainly due more to their character than to any difference in their presumed audience: The Almagest was certainly not intended primarily for readers who needed to be convinced of the necessity of the concept of the ecliptic.

In addition, Ptolemy refers as much as possible to the movements of the stars: he describes them and highlights the need to define different circles due to these movements; Theon's explanation is more like a list of definitions. Nevertheless, Theon attempts to remedy this problem by subsequently distinguishing between 'naturally given circles' or 'circles of position' and 'circles of magnitude' '°(7) This allows him to explore the rationale behind the concepts he has just defined. The equator and the tropics are given both in position and in size: they are the same circles regardless of the position of the observer·The meridian and the horizon are given in size but not in position. In fact, whether or not they are always large circles of the celestial sphere—and therefore have the same size—they are not always the same large circles depending on where you are on Earth. (2) The Arctic and Antarctic circles are given neither position nor size—since

125 Theon of Srnyrnc, *Exposition,* translated by J. Oupuis, p.215, 1.14- 19; Theon of Smyme, *Exposition,* translated by J. Delattre, p.123, (130, 1.10-15].
126 Ptolemy, 1 *'Almagesle,* translated by G. J. Toomer, I, 8, pp.46-47, H29, 1.ß-8.
127 Theon of Smyrna, *Exposition,* translation by J. Oupuis, p.217, 1..17 - p.219, 1.25; Theon of Smyrna, *Exposition,* translated by J. Delattre, p.125 [132, 1.Ii] - p.12t› [133, 1.15].
128 Theon of Smyrna, *Mposition,* translated by J. Dupuis, p. 217, 1.22-30 - p. 219. 1.1-5; Theon of *Smyrna, Expositimt,* translated by J. Oelalirc, p.125 f 132, J.6J p.126 { J32, 1.22).
129 Theon of Smyrna, *Exposition,* translated by J. Dupuis, p.219, 1.10- 15; Theon *of* Smyrna, *GxpOs Ation.* translation by J. Dclattre, p.126, [133, 1.1 -5].

The number of stars always visible varies with the observer's latitude. Moreover, Ptolemy makes no mention of this. Theon even goes so far as to distinguish between circles given in size and position relative to observers and relative to the sky: thus, he specifies that the zodiac is given relative to the sky—it is always the same circle wherever the observer is located—but not 'relative to us, because its angle with the horizon varies at the same place on Earth.'[32]

Beyond certain fairly minor differences and particular concerns that we have just noted, the various circles defined by Theon and Rolémée constitute a conceptual framework that had been the basis of Hellenic astronomy for several centuries.

t 30 Theon of Smyrna, *ExpositÎOn,* translation by J. Dupuis, p.219, 1.15-25; Theon of Smyrna, *ExpOSltlOH,* translated by J. Delattre, p.126, [133, 1.1-15].
13 l PtolCmée,£ *'Almageste,* translation by G. *I.* Toomer, II, 6, p.89
132 Theon of Smyrna, *Exposition,* translation by *1.* Dupuis, p.219, 1.5-9; Theon of Smyrna, *Expositian,* translation by J. Oelattre, p.126, [1 32, 1.23-26].

CHAPTER I: THE HYPOTHESES OF THE EARLY CHRISTIANS

Introduction

In the previous chapter, we explored the general conception of the universe held by Theon of Smyrna and Ptolemy, and the reasons behind it. We also saw what fundamental concepts served as reference points in their approach to astronomy. In this chapter, we will focus on one aspect of their astronomy: the hypotheses of the eccentric and the epicycle. These form the basis of their theories on planetary motion and are even sufficient on their own to account for the motion of the sun.

The way in which Theon and Ptolemy approach these hypotheses as geometric models is broadly similar. Both deal with them in the context of the movements of the sun, and begin by explaining how the eccentric hypothesis can account for the phenomena before doing the same for the epicycle. They then demonstrate the equivalence of the two hypotheses from a mathematical point of view.

However, Ptolemy, because he deals with these hypotheses in more detail later on as he develops his theory of the sun, is more succinct than Theon when it comes to showing in general terms that each *of them* explains "the reason for the appearance of the irregularity" of planetary movements. Theon, on the other hand, when discussing the eccentric hypothesis, wants to immediately account for quantified observations—the length of the seasons, between the solstices and the equinoxes. ([2]) Ptolemy only does this after presenting what he considers to be "theorems."

1 **Ptolemy,** ***Almagest,*** translation by G. J. Toonier, p.141.
2 Theon of Smyrna, ExyositfOfi*,* translation by J. Dupuis, p.249.

preliminary'. He then uses the same observations as Theon, primarily as data to determine the parameters for his models for the sun (in particular the eccentricity and position of the apogee). Theon presents them as observations that need to be accounted for, almost as the raison d'être of the hypotheses. I) also feels the need to account for the characteristics of the epicyclic model—which he considers more accurate—in much greater detail than Ptolemy. The latter makes his choice from the outset and simply demonstrates that it works, granting only brief remarks to other possibilities.

Furthermore, Ptolemy takes the trouble to demonstrate two points that are not demonstrated by Theon. First, he determines the positions of what would be called in the Middle Ages the "*longitudines* medine,"[(4)]to which Theon refers only very briefly and without demonstration. Secondly, Ptolemy demonstrates that in both hypotheses, the anomaly equation of a celestial body which, when viewed from Earth, appears to be at a given angular distance from the apogee, is equal to the anomaly equation of that celestial body when viewed at the same angular distance from the perigee. Ptolemy considered these results to be fundamental theorems, characteristic of the eccentric and epicyclic hypotheses themselves and not specific to the theory of the sun as such, so we will present these results in this chapter. On the other hand, the direction of the apogee, as well as the eccentricity of the solar orb, although the demonstration is based on the same observations (and almost the same figure) as those used by Theon in his demonstration of the validity of the eccentric hypothesis, will only be dealt with in the following chapter because they are parameters specific to the theory of the sun.

We will therefore begin by presenting Theon's hypotheses, to which he devotes particular attention. In the second part, we will present his demonstration of the validity of the eccentric hypothesis, followed by that of Ptolemy. We will then do the same for the epicycle hypothesis (in

3 Pioléméc, *f'Alma(este,* translation by G.J. Toomer, IH.4, p.153 [H232. 1.2].
4 O. Pedersen, *A Survey of the Alma(est,* p.141.
5 Angular difference between the average position and the observed position.

thirdly), then, in a fourth part, to demonstrate the equivalence of these two hypotheses proposed by each of the astronomers.

I - Presentation of theyh otheses in Theon

Theon first presents the data he intends to explain using two hypotheses. He characterizes the apparently irregular movement of the sun in two ways: by the different lengths of the seasons (a function of the average angular velocity of the sun in a given quadrant of the ecliptic) and by the speed of the sun at different points on the ecliptic. He places the spring equinox at the beginning of Aries without further precision, and does the same for the autumn equinox (at the beginning of Libra) and the solstices (the winter solstice at the beginning of Capricorn and the summer solstice at the beginning of Cancer). We can assume that Theon's lack of precision was motivated solely by a desire to simplify his explanation. 1) indicates the time taken by the
sun in days to travel from each of these points to the next, summarizing this information in the following figure:[7]

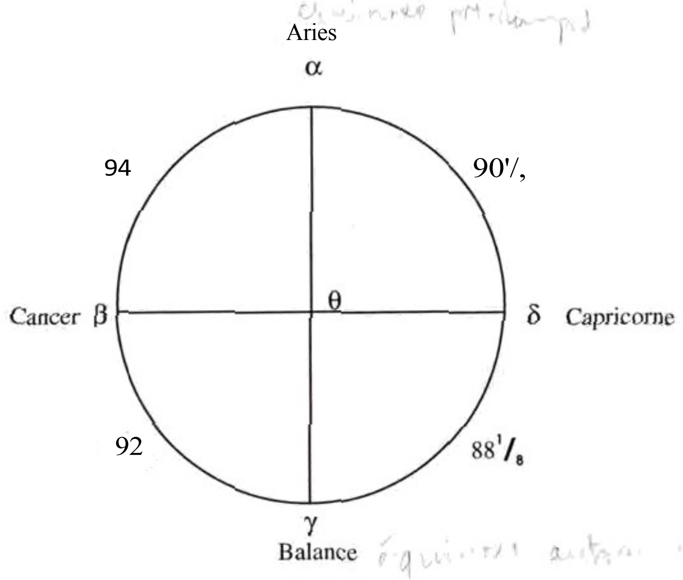


Fig. 2.1

6 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p. 249, 1.1–3; Theon of Smyrna, *Exposition,* translation by J. Delattrc, p.149, [153, 1.2-5].
7 Thöon of Smyrnc, *Ef›osifiarf,* translation by /. Dupuis, pp.34g-249; Thënn of Smyrne, €zyeïyïoa, translation by 1. Dclattre, p.149.

The sun therefore takes:

from the spring equinox to the summer solstice: 94'/ days

from the summer solstice to the autumn equinox: 92[1] ft days

from the autumn equinox to the winter solstice: 88'f days

from the winter solstice to the spring equinox: 90 li days

This makes a tropical year of 365.25 days.

Theon then indicates that the speed of the sun is minimal when it enters Gemini, maximal when it enters Sagittarius, and average in Virgo and Pisces. This is all the data he will now attempt to account for.

In presenting his two hypotheses, Theon attempts to give them a character of necessity. He derives them naturally from considerations based both on the data he has just presented and on the idea that the stars, by their divine nature, must be animated by a "uniform and regular" motion.[0] as required by the principle laid down by Plato. Starting from this latter principle, if the earth were located at the center of the sun's orb. We would see it moving uniformly , i.e. with constant angular velocity — on its circular orb. Since this is not the case, Theon concludes that the earth, or rather its center Õ- (and therefore that of the universe), is not located at the center of the orb.[11] In such circumstances, he distinguishes three possible cases. Either O- is located inside the circle along which the sun moves, or it is located on this circle, or it is outside it. (2)

8 Theon of Smyrna, ***Exposition,*** translation by J. Dupuis, p.249, **1.1 -3; Theon of** Smyrna, *Exposition,* translation by J. Oelattre, p.149, [153, 1.6-12].
9 Theon of Smyrna, *Exf›osilion,* translated by J. Dupuis, p.249, 1.10-12; Theon **of Smyrna, *EKpŒfÙ/Hn,*** translation by J. Delaitrc, p.149, (I53, 1.12- l4J.
10 Theon of Smyrna, Mposifioit, translated by J. Dupuis, p.249, 1.15; Theon of Smyrna, *Myosition,* translation by J. Delattre, p. 150, [t 53. 1. i g1.
l 1 Theon of Smyrna, *Mposition,* translated by J. Dupuis, p.249, 1.1 S-p.251, 1.3; Theon of Smyrna, *Mposition,* translated by J. Dclattre, p.150, [153, 1.20- 154, 1.4|.
| 2 Theodotus of Smyrna, *Exposition,* translation by *1.* Dupuis, p.251, 1.3-5: Theodotus of Smyrna, *Exposition,* translation by J. Delaitre, p.150, (156, 1.5-6}.

Il eliminates from the outset the second possibility because in that case, he says (probably quoting Adraste):

> ...the Sun itself would be present on Earth, and for some people on one side of the Earth, it would always be day, while for others, it would always be night, and the sun would neither appear nor rise nor set, nor move around the Earth at all.

The meaning of this passage is unclear. It could mean that the *sun* would always remain "present on the earth" and appear motionless, but in that case it would no longer make sense to speak of a solar orb or simply of the movement of the sun, since the earth is motionless for Theon. In this case, the sun would also be completely motionless. The passage must therefore be interpreted as meaning that the sun would be "present on earth" once a year, and that although it moves along its orbit, it would only be visible from half of the earth, completing a journey across the visible part of the sky in one year, without ever appearing to disappear behind the horizon.

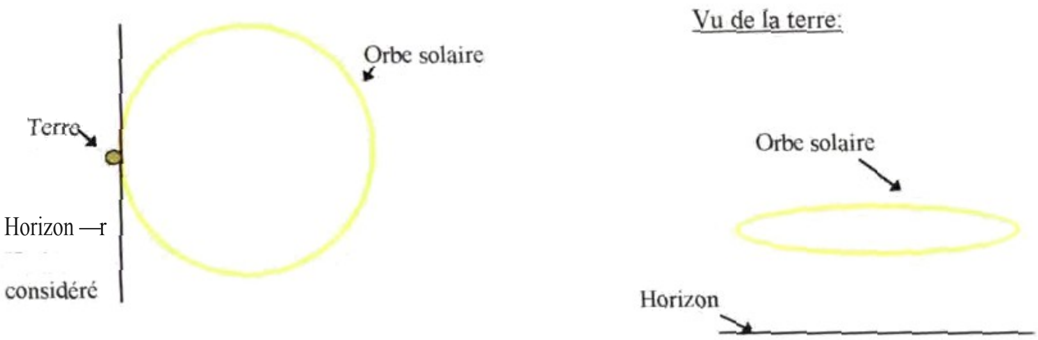


Fig .2.2

13 Théon de Smywe, *Expoûtion,* translation by f. £klattrc p 1$ti| ls4. 1.8-11]: the translation that J Oupuîs gives of this passage is as follows c so il renconireraît 1 erre ‹kint es hab tan s aura ri les uns toujours le jour, les autres toujours la nuii: there would be no rising or setting, and no one would believe that the sun revolves around the earth, which is absurd." Theon of Samos. M "a". Translation by J. Oupuis, p.2$1. 1.6-10).

But even this interpretation presents difficulties. On the one hand, even if we still consider the annual movement of the sun, there would be places where the solar orb would pass belowhorizon. Admittedly, even in ces places, the phenomena would not correspond to those observed because the sun would not appear to make complete revolutions relative to the fixed stars: it would appear to move back and forth across constellations that would represent significantly less than B0° of the sphere of fixed stars. One might therefore think that Theon (and Adrastus) simply neglected these cases because they do not correspond to the phenomena either.

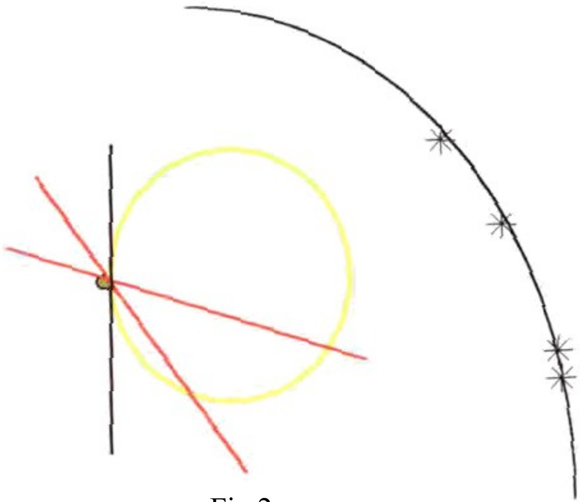


Fig.2

.3

More troubling, however, is that the sunrises and sunsets to which Theon refers do not depend on the movement of the sun in its orb, and therefore do not depend on the position of the Earth relative to it either. In Theon's vision, they are due to the rotation of the sphere of the £xes, which causes the movement of all the stars—including the sun. The center of this sphere corresponds to that of the Earth, not the solar orb. The fact that it passes through the Earth should therefore not prevent the sunrises and

ce dernier sunsets from occurring.

the other, except once a year:

dalle ... stern ... orb

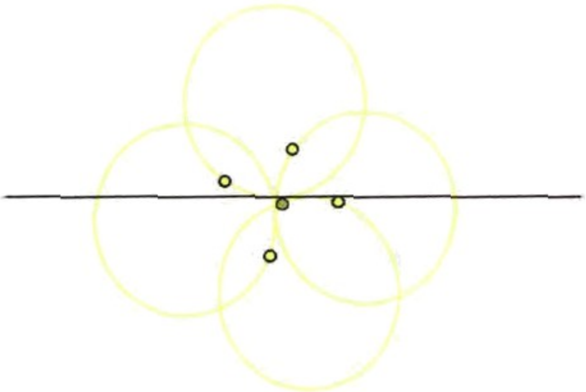


Fig. 24

In any case, it is clear that Theon considers the idea of an orb passing through the earth ridiculous, and that he expects the same reaction from his readers. He therefore considers the two other possibilities: that O• is either inside the solar orb—without being at its center—which corresponds to the eccentric hypothesis, or outside the circle on which the sun moves, which corresponds to
ä the epicycloid hypothesis. He already states that both possibilities explain the appearances, but this is a subject that we will discuss in detail below.

II - Hypotbèse de l'ezcenfrque

A - In Theon

Theon therefore begins by demonstrating that the eccentric hypothesis allows us to *explain* the apparently irregular movement of the sun, and in particular the observations he mentioned above. To do this, he bases his argument on the figure:[15]

.253, l.1 - p.255, l.28; Théon de Smyrne. .1 [157, l. .

Expa iiion, translation by J. Delattre, p.151 (US, 1,1) - p 53 1: }

}5 Tbdo0 by **Siayrae, Wj** osii'ion, **translated by** J, **Dupuig p.252;** Tftdon by Szrgmze, too. translation by J De tre 32

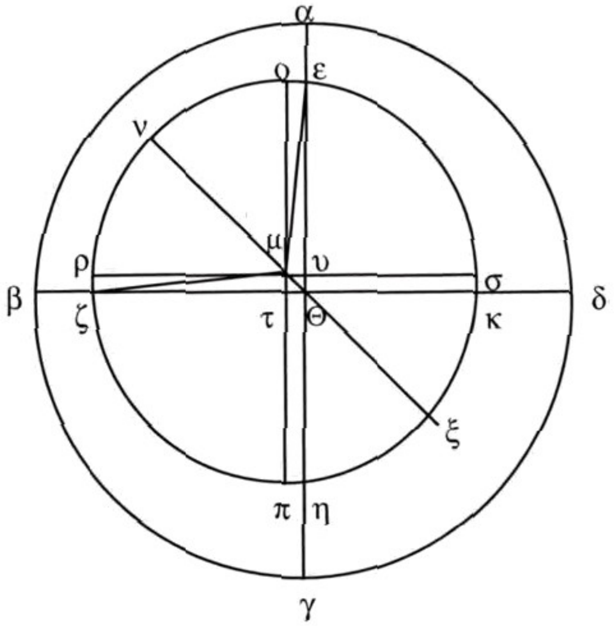


Fig.

In this figure, the outer circle n§y6 represents the ecliptic and the circle opnx represents the orbit of the sun. This is an eccentric circle since its center is at point (t, while the earth, being at the center of the universe, is in the middle of the ecliptic, i.e., at point O-.

Theon begins by using only the following elements of the figure:

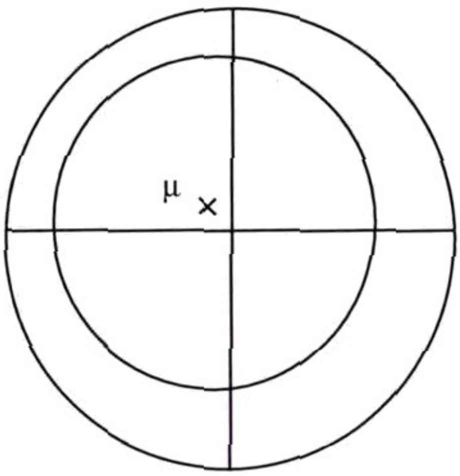


Fig.2.6

Following the same goal set by Greek astronomers since Plato and Aristotle, Theon wants to show that the sun moves in a uniform motion. The eccentric must therefore be such that the luminary travels through each of its quadrants (op, pu, zo, and po in Fig. 2.5) in equal times. Seen from Earth, where the arcs of the ecliptic n§, §y, y6, and 6o are actually seen as equal, the sun therefore appears to travel through quadrants by traveling through the arcs eÇ, ÇT}, r{x, and xE, when in fact none of these arcs is a quadrant of the solar otbe. The star therefore travels through them in different times, while appearing to cross equal arcs on the ecliptic.

In order to deal with the problem quantitatively, Theon immediately translates the figure into terms that will allow him to talk about the length of time during which the sun appears to travel through the different arcs. He thus chooses to divide the eccentric into 365'/‹ parts so that each of the divisions is covered by the sun in one average solar day.[6] In this unit, each quadrant of the eccentric measures a little less than 9 l '/3 (91 / t), the number of days that the sun would spend in each quadrant of the ecliptic if its movement were uniform as seen from Earth , which Theon only specifies later." In order to obtain the observed data, he assigns numerical values to each of the arcs of the eccentric (eÇ, Çr{, ipt and xE) which, when viewed from Earth, correspond to quadrants of the ecliptic. Thus, to eÇ, the largest of these four arcs, he assigns a measurement of 94'/ of the units in question, because the sun travels this arc in 94'/i days, which corresponds to the time it takes for the star to cross the quadrant of the ecliptic that runs from the spring equinox to the summer solstice—i.e., the duration of spring. Similarly, he attributes a measurement of 92'/ to §r{, to T}R 88'/g and to xt 90'/g," O thus showed that the hypothesis of a circular but eccentric solar orb allows us to account for the fact that the sun appears to travel through the quadrants of the ecliptic defined by the equinoxes and solstices i.e. the

] O Theon of Smyrna, *Mposilion,* translation by J. Dupuis, p.253, 1.4-5; Thcun of Smyrna, *Exposition,* translation by J. Dclaitre, p.151 [155, 1. t -4].
f 7 Thcon *of* Smyrna, *Exposition,* translation by *I.* Dupuis, p.257. 1.3-4; Theon of Smyrna, *Exposition,* translated by J. Delattre, p.l 54 [157, **1.18-20].**
18 Thüun de Smyne, *Exposition,* translation by J. Dupuis, p.25.3. 1.5-G; Théon de Smyrne. *Exposition,* translation by J. Oclattrc, *p. t5l* [f55, 1.4-fil.

seasons — in different lengths, equal to 94'/t, 92'/i, 90'/t, and 88'lt days, while traveling through its orbit even with what we would describe today as a *uniform* "angular velocity.″

Tliéoii then shows that this hypothesis allows us to account for another 'fact' related to the maximum and minimum speeds of the sun on its orbit, which he had mentioned at the beginning as data. 11 partially completes the figure *by* adding the straight line passing through the centers of the ecliptic O and the eccentric (t, which allows him to define what we call the 'apogee' and the 'perigee' (he does not use these terms), respectively at v and §.°° He explains that when the sun is thus at its greatest distance from the earth, it seems to us to move the least rapidly,

phenomenon that occurs in II 5' 2. Conversely, the 'perigee' is located at ä'/2". It is hardly surprising that the model incorporates the maximum and minimum speeds of the sun, since it was developed for this purpose. in reality, Théon simply assigns the appropriate locations on the ecliptic to points v and § of the model, knowing that, according to the model, these two points are those where the sun must move at its lowest and highest speeds, respectively. Theon is well aware of this, as he hastens to provide 'data' that he had not previously mentioned. This data corroborates the model, as it means that the sun is actually at its greatest distance from the earth when it appears to be moving the slowest — and vice versa. Indeed,

Theon indicates that when it is at point v, the star "will appear smallest in size" and that the opposite will occur when it is at §.[2]However, he remains cautious, stating that this is what the sun " " "seems" or "appears" to do at fi 5't2 and J 5'/;

19 Theon of Smyrna, Eryosifiozr, translation by J. Dupuis, p.253, 1.4-p.254, 1.16; Theon of Smyrna, *Exposyion,* translation by J. Dclattre, pp.152-153| 155, 1.5 - 156, J.21].

20 Theon of Smyrna, *Mposilion,* translation by *1.* Dupuis, p.255, US-1.17; Theon of Smyrna, *Exposition,* Translation by J. Dclattre, p. 153 [l 56, 1.22 - 157, 1.2].

21 Theon of Smyrna, *Exposition,* translation by J. Dupuîs, p.255, 1.18-25; Theon of Smyrna, *Mposiii* , translation by J. Delattre, p.153 [157, 1.2 - 1.9].

22 Thcon of Smyrna, *Exposition,* translation by J. Delattre, p.U3, p| 157, 1.3-4]. Or 'when it appears to us to have the minimum size', Theon of Smyrna, *Er,nosirinn,* translation by J. Dupuis, p.255, 1.19-20.

23 'it will appear to have the maximum size', Theon of Smyrna, *Mposilion,* translation by J. Dupuis. p.255, 1.22-23; where 'it will seem closer to the earth', Theon of Smyrna, *Expo.*iation, translation by J. Dclattre, p.153, [157, 1.6-7].

respectively. He will subsequently set these doubts aside *and* assume that !"apogee" corresponds to the minimum speed.

Theon deduces from these considerations that the sun has an average speed and size when it is halfway between these points, at m 5'/t and @ 5'/,'[4] His conclusion is most certainly due to the fact that the sun is then at an average distance from the earth according to fig.5. However, not only does *Theon* fail to develop a demonstration, but he introduces the idea with some uncertainty, using the expression translated as "...if we are right, it (the sun) appears *to have* an average size and speed..." ([2]) "it is certainly logical that this is the case..." ([26])

B - **In Ptolemy**

As we mentioned in the introduction, Ptolemy deals with the validity of the two hypotheses in a more abstract way than Theon, not immediately attempting to account for quantified observations, which he reserves for the theory of the sun itself. Nor does he seek to present the two hypotheses as necessary. He gives the impression that other hypotheses could, after all, also be considered. He therefore describes the two hypotheses from the outset and immediately sets out to demonstrate that they can account for the apparently irregular movements of celestial bodies, which are in fact uniform movements.

Ptolemy also begins by discussing the heliocentric hypothesis. However, the way in which he defends the idea that it accounts for the apparent movements

24 Theon of Smyrna, *Exposition,* translated by *I.* Dupuis, p.2'î5, 1.25-27; Theon of Smyrna, ***EXPOSITION,*** translation by J. Delattrc, p.153 1 57, l.9-11].

25 Theon of Smyrna. *Exposition.* Translation by J. Theon of Smyrna, *Kxpnsitfun,* translation by J. Dupuis, p.25'î, 1.25-26.

2ö Thëon of Smyrna, *Exposition, translation* by J. DrJaiirc, p.15?, *l* 1.57, 1.9J.

The irregularities of the sun are different from those of Theon. They are also simpler, as can be seen from the[27] following figure:

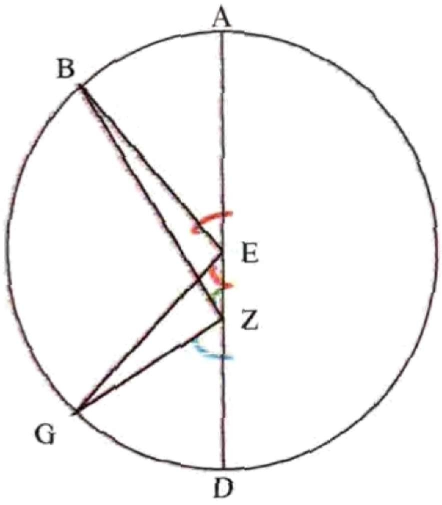


Fig.2.7

In this figure, the circle represents the eccentric and E is its center, Z being the center of the ecliptic (which is not shown) and therefore the Earth, the position of the observers. He chooses two equal arcs seen from E, AEB and GED, measured one from the apogee A, the other from the perigee D, manière so that só that qu'il veut the phenomenon

becomes as evident as possible. The sun therefore travels along these arcs in equal times. However, seen from the Earth, when moving from A to B, the sun appears to have moved by the angle AZB, which is less *than* AEB, *and* when it moves from G[28] to D, it is seen to travel through the angle GZD, which is greater than GED; it thus appears to travel through unequal angles in equal times. . Il

Rolémée stops there in his general demonstration that the eccentric hypothesis can account for apparently irregular movements, simply stating that in the case of this type of eccentric model, the other can only reach its minimum speed at apogee (and maximum speed at perigee). He does not show

27 Ptolémée, l'Almageste, traduction de G.J. , III, 3, p. ; Fig.3.1.
28 Ptolémée, l'Almageste, i .J. .6-15].
29 Ptolémée, l'Almageste, .J. .11-13].

eo . . : . -
translation by G T‹x›mer. III. 3; p.14-t [H218. 1

more detail than the eccentric hypothesis can account for the phenomena than after discussing the epicycle hypothesis in general terms.

The fact that Ptolemy's demonstration differs from that of Theon is most certainly due to the fact that he develops the theory of the sun further than Theon does. Ptolemy is familiar with Fig. 2.5, which he refers to a little later when he wants to determine the parameters of the model of the sun—in this case, the eccentricity of its orbit.[30] If he does not use it immediately, preferring to give the demonstration we have just seen, it is certainly because the latter is simpler.

One might therefore wonder why Théon demonstrates the validity of the eccentric hypothesis based on Fig. 2.5. I) It is likely that this was also done for reasons of economy. Not wishing to dwell too long on the subject, but nevertheless wanting to show how the length of the seasons is accounted for at least once (he does not do so in the context of the epicyclic hypothesis), since it is the observations themselves that he presents as requiring an explanation, he may prefer to merge the two demonstrations. He can do this by reasoning directly from Fig. 2.5, which is a more complex version of Fig. 2.1, in which he represented these data—the points ri, {}), y, and ô correspond to each other in both figures. On the other hand, it would be difficult for him to do this by comparing the angles seen from the earth with those seen from the center of the eccentric.

It should be noted that Théon does not actually use any figures indicating these angles anywhere. nor the angle of the equation of anomaly. This is most likely because, not wanting to go into detail, he does not feel the need to do so. It is so natural for anyone who wants to determine the positions of the sun based on the eccentric model to use such *figures and* to reason from these *angles* that we can assume that this is what Hipparchus (and perhaps even Adrastus) did, especially since Ptolemy repeatedly acknowledges his debt to him.

3fJ Ptnlémée. *l'Almagesle,* translation by G.J. Toomer, III, 4; p. t54, Fig.3.9.

After already expressing the opinion that the two hypotheses are mathematically equivalent, but before proposing what can be considered proof of this equivalence, Ptolemy determines the positions where the star reaches its greatest anomaly and its apparent average speed—the "*longitudes mediae.*" He demonstrates that from *Earth*, these points appear to be located at 90' from the apogee or perigee. In this respect, he agrees with Theon, who places the positions of the sun's average velocity at G S'f and @ 5'li "' , i.e. at 90º from the positions fi 5'/, of the apogee and 5' of the perigee of the solar orb. However, Theon does not provide any proof for his assertion. He even seems to consider it as observational data. Furthermore, he does not specify that these points correspond to the maximum anomaly. That said, what Ptolemy actually denies is that the points of maximum anomaly are located at 90º from the line of apsides. However, he does not demonstrate that these are the locations where the apparent speed of the sun corresponds to its average speed. As Toomer points out, "it is intuitively seen to be true from the epicyclic model."

Ptolemy reasons from the following figure:

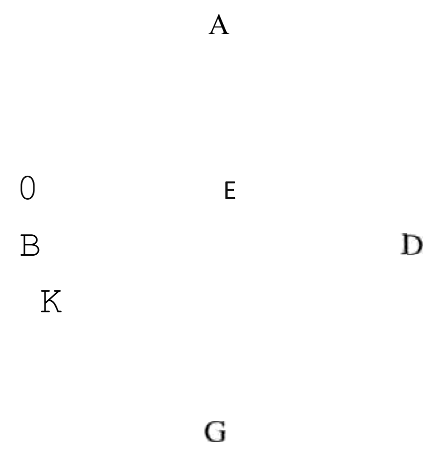


Fig. 2.8

3 l Ptolëmce, f *'Almaze.mtv,* translation by G.3. Toomer, lIl,3, pp.146- 1 47 {H22 1, l,fi - H223, 1.7}.
32 Thfon of Smyrna, *£xpoxitlun,* translation by J. Dupuis, p.2fi5, L 25-27; Thèon *ôe Smytnc, ExF o›* itiofl, translated by J. Delaltre, p.1 'i3 { 157, 1.9- 11].
33 droiic joining the apogee and the perigee.
14 G.J, Toonicr, *Ptolem¡• 's Almay:ext,* p, 146. note J5.
35 Piolcmée, *1 'Almuzeslr,* translation by G.J. Toortier, l II, 3, p 14s, f-ig.?.3.

The circle again represents an eccentric orb, with point E as its center and Z as the center of the ecliptic (the earth). He chooses B and D such that BZ and ZD form right angles with the line of the apsides, and takes O- and K as any points on either side of B. The difference between angles AEB (which marks the angular position of the sun when it is at B, measured from its apogee and seen from the center of its orb) and AZB (which marks the angular position of the sun when it is at B. always measured from its apogee, but seen from the earth) is the equation of anomaly of the star when it is at B. Similarly, the difference between angles AEO- and AZO-, and that between angles AEK and AZK, represent the equations of anomalies of the sun when it is *at* O- and K respectively. Rolémée must therefore demonstrate that the difference between angles AEB and AZB is greater than the difference between AEO- and AZO-, and that between AEK and AZK. wherever O and K are located on either side of B. To do this, he notes that the difference between angles AEB and AZB is equal to angle EBZ. This s'can be explained as follows—which Ptolemy does not detail:

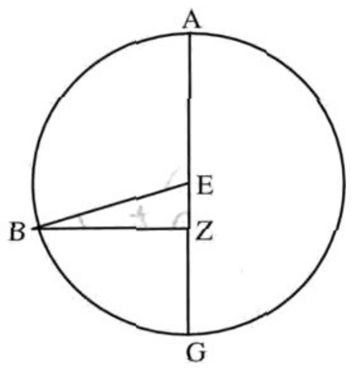


Fig.2.9

AEB+ BEG= 180‘

EBZ + BZE+ BEG= 180º (sum of the angles of a triangle)

d'où: AEB= EBZ+ BZE

AEB= EBZ+ AZB

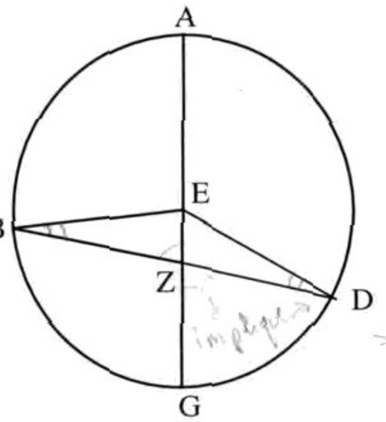
AEB - AZB= EBZ

equation d'anomalie à B.

The fact that the differences between AEO- and AZO, and between AEK and AZK are equal to the angles EO-Z and EKZ, respectively, is not explicitly stated, but this goes without saying, since the reasoning does not depend on the fact that the angle AZB is straight. This is an equivalence that will be very *common* in the text of *the Almagest. The* problem now returns to demonstrating that the angle EBZ is greater than any angle EOZ or EKZ, which it does through geometric reasoning.

Ptolemy also concludes that angle AEB is greater than angle BEG by a value equal to twice the anomaly equation. The time interval between the moment of minimum velocity and that of average velocity is therefore greater than the time interval between the moment of average velocity and that of maximum velocity.[36]

Ptolemy presents one last point as part of his "preliminary theorems," namely that the equation of[37] anomaly is the same when, as seen from Earth, a celestial body appears → d' B et a D
to be at the same angular distance from the apogee and the perigee. In other words, '... the arc (...J of mean motion counted from the apogee A will exceed the arc of apparent motion [...], by the same equation [...] as the arc of mean motion counted from the perigee G is exceeded by the arc of apparent motion.' 11 demonstrates this again in the context of the two hypotheses. As for the eccentricity, his reasoning can be summarized as follows.[39]



→ même equation d'anomali

Fig. 2.10

.36 Ptotcmée, *l'Almageste,* translation by G.J. Toomer, III,1, p.147 [H223, 1.3-7].
37 Ptolemy, / *'Almaffesle•,* translation by G.J. Toomer, III,4, p.153 [H232, 1.2].
fi8 Ptolemy, / fi/muye+zr, *translation* by G.J. Tourner, III,3, p.152 {H230, 1.8- 11 J.
39 Ptolemy, */'Almugesie,* translation by G.J. Toomer, I II,3, pp.15 l -152 {H229, 1.l4-H230, 1.1 3J,

The angles AZB and **GZD** are equal since we are considering situations where the star is seen from Earth at angular distances equal to the apogee and the epicycle. This means that B, Z, and D are on the same straight line. The triangle BDE therefore has angles B*,* ZBE and D, ZDA. Since the triangle BDE has equal sides BE and DE (these are the radii of the circle), its angles ZBE and **ZDE** are also equal. Now, these angles are the equations of anomaly.

This is neither demonstrated nor even mentioned by Theon. This is most likely due to the fact that this information is mainly useful for developing a precise theory of the movement of the sun, which allows its position to be determined at different times, as Ptolemy does; but it is not essential *for* a general understanding of the hypotheses.

tII hH èmde e ee

A - In Theon

After discussing the eccentric hypothesis, Theon goes on to demonstrate that the epicyclic hypothesis also accounts for the phenomena. For the former, he had chosen from the outset that the direction of the sun's revolution on the eccentric was from west to east, in accordance with its movement relative to the ecliptic. This time, he distinguishes several cases. He first determines the direction of revolution of the epicycle, then, once this has been decided, he considers the direction of revolution of the sun on the epicycle.

The cases that Theon distinguishes for the movement of the epicycle are as follows: either it is stationary relative to the earth, or it moves in the same direction as the sphere of fixed stars, or it moves in the opposite direction. In the first case, the sun would neither rise nor set, and it would be permanently daylight for half of the earth and night for autumn [4] Theon therefore rejects this possibility fairly *quickly.* He does not study the case of an epicycle moving 'in the opposite direction to the universe', but merely mentions that this is also a possibility, and that it is the one preferred by Plato. [41]

He therefore focuses mainly on the case where the epicycle moves in the same direction as the sphere of the fihes,[42] and again distinguishes several cases: either the epicycle has an 'angular' speed equal to that of the 'universe' (J= cy),[4] or it is greater (>) or less (w< tu)." Theon rejects the first case, because then the sun would appear to move back and forth along part of the ecliptic, but would never complete a full circle. He explains this carefully with the help of the following figure:[45]

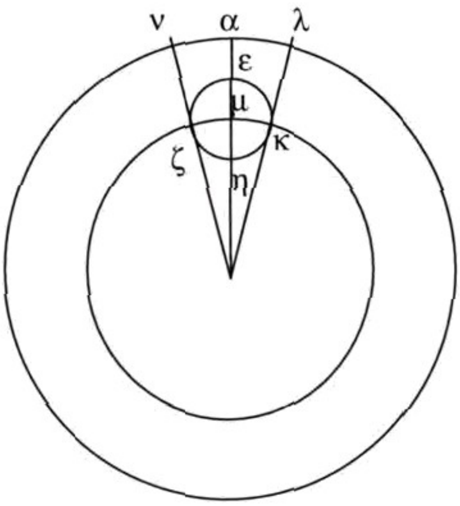


Fig. 2. 11

0 deals more succinctly with the case where > ny, explaining that in that case the sun would appear to move eastward across the ecliptic [46]

40 T duri *de* **Smyrn,** ***Eposiiiori,*** **translation by** *1.* Dupui.s, **p.2'î7, I.21-28: Thêon** dc **Smyme** , *Exposition,* **translation** by **1. Dclattre, p.** I "**îS**| **139,** I.7-{]].
41 Theon of Smyrna, *Exposition,* translation by e J. Dupuis, p.261, I.11-13; Thëon of Smyrna, *E.rposiiion,* translated by J. Dclattre, p.157 [l fi0, 1.18-l9|.
42 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.259, 1.1 267, I.32; Theon of Smyrna, *Mpo.sition* translation by J. Delattre, pp.156-161 [UO, 1.5-1 GS, 1.12].
43 Setting tg: angular velocity of the epicycle and ig: angular velocity of the sphere of the rxes.
44 Theon of Smyrna. *Mposiiion, translation by 1.* Dupuis, p.259, 1.3-4; Theon *of* ***Smyrna,*** *Exposition,* translation by J. Delaltre, p.l55 [159, 1.14- 16].
45 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.259, t.3- 14; figure p.25R.
46 Theon of Smyrna, *Mposition,* translation by *1.* Dupuis, p.259, 1. 14- t7: Theon of Smyrna, *Exposition,* translated by J. Delattre, p.156 {160, 1.1-4) .

Theon concludes that the situation as described by< ny is therefore the one that can account for the phenomena, since the sun thus appears to be 'left behind' and to travel across the zodiac from west to east·

11 He still has to determine in which direction the sun moves on the epicycle. Before any demonstration, he posits that the luminary travels around the latter in one year. [4]That is to say, with the same 'angular velocity' as the epicycle relative to the fixed stars.

Theon then shows that the sun cannot rotate on the epicycle in the opposite direction to the sphere of the fixed stars, i.e. from east to west. To do this, he reasons from the following figure: [0]

Fig. 2.12

When the epicycle is at (t, the sun is at e. Since the star is then at its greatest distance from the earth, the corresponding point on the ecliptic, Cf, is fi 5 lt. Conversely, the point y corresponds to ' 2 When the epicycle passes from [i to o (or from § to p) — which takes place in one period — given the chosen direction.

47 Theon of Smyrna, *Exposition,* translated by J. Dupuis. p.259, 1.18 p.261, 1.5; Theon of Smyrna. *Eryosiiion,* translation by J. Dclattre, p. t 5ô { 160, 1.5-12].

48 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.261, 1.1 3-14; Theon of Smyrna, *Exposition,* translation by J. Dsfattre, p. I57 *l6G, 1.*19-20].

49 Theon of Smyrna, *Exposifion,* translated by J. Dupuis, p. 263, 1.5-p. 265, 1.10; Theon of Smyrnc, *ÑWflO.S* SHOft, translated by J. Delattrc, pp.157-159 [161, l.3-162).

5t) Theon of Smyrna, *Exposition,* translated by J. Dupuis, p.260.

for the rotation of the sun on the epicycle, the other moves from e ä ri (or from to e).'" I) therefore seems to describe the arc no (or tom), which represents more than one quadrant of the ecliptic. Following the same reasoning, we see that the sun takes the same time—i.e., the quantity of a year—to describe the arc œy (or), which represents less than one quadrant of the ecliptic. fl therefore moves faster when it is in II ' 2 than when it is in 5'/"which is contrary to observations. This conclusion is based on the idea that the point on the ecliptic where the angular velocity of the sun is minimal, H 5'/", must necessarily correspond to the "apogee" of the orbit (this is what allows us to identify it with H 5'/2 and to compare the hypothesis with observations).

Thdon examines the opposite hypothesis, *according to* which JaqueNe Je soJeiJ moved on the epicycle in the same direction as the sphere of fixed stars rotates."[2]He reasons from a figure similar in principle to the previous one, which he simply adapts to this new situation:

Fig. 2.13

5 l The movement of the astrc on the epicycle is counted from the upogec of the latter, i.e. from the point on the epicycle furthest from the earth, i.e. the two points of intersection between the epicycle on the one hand and the line passing through the Earth and the center of the epicycle on the other. Here, this point is e when the epicycle is at p, k when it is at o, and t when it is at (.

52 Theon of Sinyrne, *Exposition,* translation by J. Dupuis, p.265, 1.1 1-p.267, 1.32; Theon of Smyrnc, *Mposiiion.* Translation by J. Dclattre, pp. 159–161 [163, 1. 1–165, 1.12].

53 Thcon dc Sttiyrne, *Exposition,* translation by J. Dupuis, p.264, Thcon dc Smymc, *Exposition,* translated by J. Delatire, p. f fi0.

Following the same reasoning as before, we see that this time the arc no (or u›ci) that the sun appears to travel in a quarter of a period is less than a quadrant, and conversely where (or ym) is greater than a quadrant of the ecliptic. Theon deduces that the sun appears to move more slowly in ct than in y, which is consistent with the facts. Again, this conclusion is based on the idea that point ‹x, the apogee of the solar orb, corresponds to 5'/,.

Furthermore, this time Theon also offers an interpretation 11 explaining that when the epicycle moves from o to v and then to f, the movement of the solel on this epicycle is such (from X to D and from u to x) that it is added to that of the epicycle—and, conversely, subtracted from it during the other half of the period.

Theon thus determined that the epicycle hypothesis accounted for the phenomena provided that either the epicycle moved in the same direction as the sphere of the fixed stars but with a lower angular velocity, or that it moved in the opposite direction and at the same time the sun moved in the direction of the sphere of the fixed stars, which can be represented as follows:

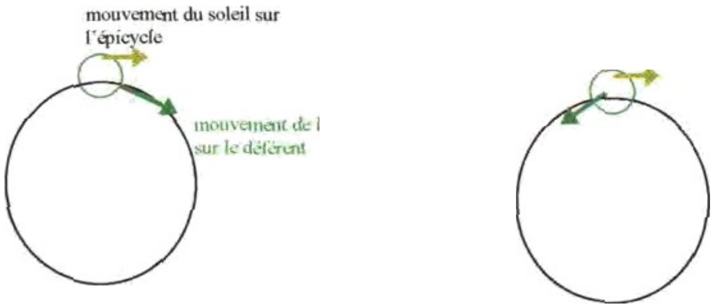


Fig. 2.14

11 It should be noted that, contrary to his demonstration concerning the eccentric hypothesis, Theon does not bother to interpret the epicycle hypothesis in numerical terms: he does not explicitly specify the positions of the epicycle and the sun corresponding to the different seasons. However, the answer to this problem is in

54 Theon of Saivrne, *Mposition* translation by *1.* Dupuis, p.267. 1.23-32: Theon & Smvrne, *k-"xposiiiun.* translation by J. Delattre, pp,l6tï-161 [165, 1.2-12].

some kind of answer is implicitly given by the fact that he identifies point ct with H 5'/t. and will be even more so when he demonstrates the equivalence of the two hypotheses in the following paragraphs.

B - **Che:Z Ptolemy**

The way in which Ptolemy demonstrates the ability of the epicyclic hypothesis to account for irregular movements in terms of uniform circular movements differs from that of Theon, but less so than his demonstration relating to the eccentric hypothesis. On the other hand, unlike Theon, Ptolemy is extremely brief—even more so than when he deals with the eccentric hypothesis. Ptolemy uses Fig. 2.15, in which the circle **ABGD** represents the deferent (of course concentric in this case), and the circle ZHO-K represents the epicycle.

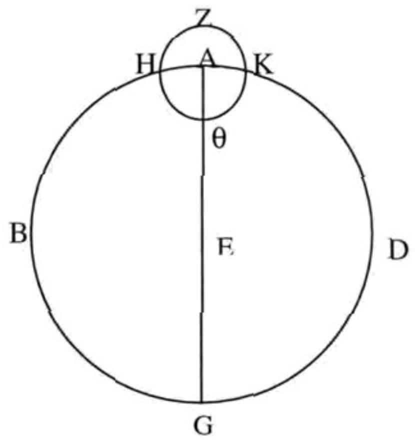


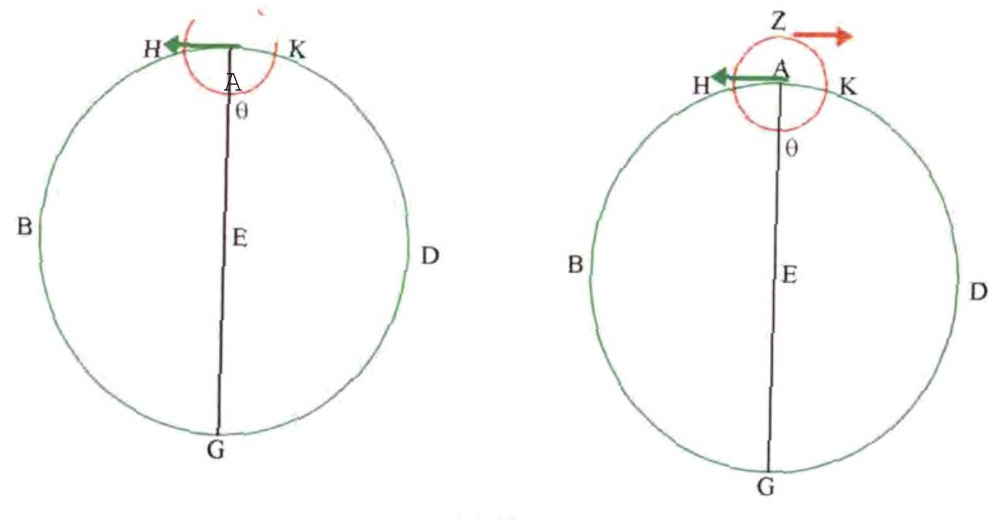
Fig. 2.15

At this point, Ptolemy simply notes that, although the star moves uniformly on the epicycle and the epicycle rotates uniformly on the deferent, it is only when it is in positions Z and O- that the star appears to be in the same place as the center of the epicycle, whose movement appears uniform.

55 Ptolèniüc. *l 'Ähnagesle,* translation by G.J. Toomer, p.145, Fig. S.2.

meseen fromtheearth. Otherwise, •'l **PP t qul the epicycle cturnsfromAtowards if the star is, for example, at H, it will be seen ahead (of) (center) (of) (the epicycle) (and) (in) âfrÎère if it is in K,"

ROlémée simply states that in the case of the *epicyclic* hypothesis, c unlike the eccentric *case*, the celestial body can have its maximum velocity both at its apogeeand at its perigee, depending on its direction of revolutionionthe(epicycle). (" If) (to) (u) (rne) from west t o east (from Z to H), t h e maximum speed (will be) (reached) (at) (the) (apogee,) PWCe Ethe starse movedC!fS ddD8 lg same direction;° sj p2tp co tre it turns from east to west (from Z to K), its speed will be minimal at apogee since it will be moving 8lllSlinthe direction opposite to that of the epicycle." We cansreprésenter the SltttâtlOn as follows:



big.2 16

56 Ptolémée, *l'Almageste*, traduction Ü.J. ToolR0F, III, 3, p. 144 [H218 I.J-7].

57 Ptolémée suppose que l' correspond effectivement aux modèles 3, p.144 [H218, I.14-H219, c t°ume d'Ou in Gsl. e‹ ccci *l'Almageste*, traduction de G.J. Qu'il Va développer

58 Ptolémée, *l'Almageste*, traduction de G.J. oomer, III,

59 Rolémée (pitt pqjp traduction de G.J. Toomer, III, 3, pp.144-145 [H219, 1.1-4]. ôuprincipleque* ePicycle tour»• from West to East On the deferent.

60 Ro}élflée. *l'-4Ima*$_{ge}$ *ste,*

61 Rolém8e. *l'-4 Imaging,* Ofl by G.J. T omer, 3. p 145 [H219. 1.4-i .

Within the epicyclic hypothesis, Ptolemy determines the positions of maximum anomalies (and average v-values) using Fig. 2 17, reasoning first from what is shown here in black. The circle EZH is the epicycle and ABG is the deferent (whose center D therefore represents the Earth)

E

G

! H

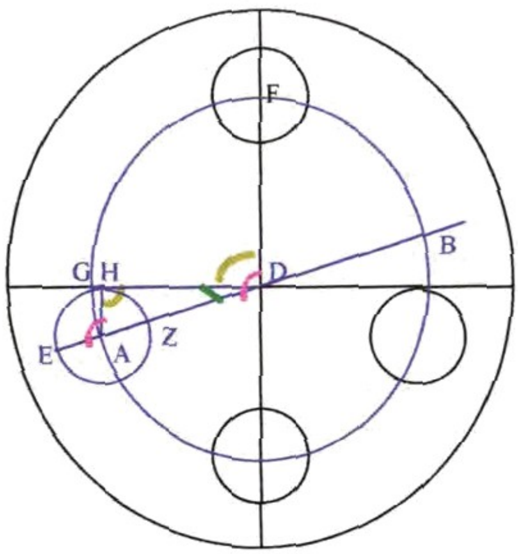
D

Fig.2.17

The position of the epicycle represented in this figure corresponds to what is shown in blue in the diagram below (which Ptolemy does not provide), i.e., in the case where the star, at H, is seen from Earth (D) at a quadrant of its apogee.

f›2 Rolcnicc. / lfi» zcz•.i'ie. translation by G.J. Toomer. 111. 3. p. 144. Fig.3. 4.
L›”› RoTñf»ec.. / I//» ›,¿‹•. fu. Translation by G.J. Toomer. I II, 1. p.]17] H?21. I.8-H*24. I]‹›{,
f›t RaTcn ñc. / 1/»tzi,vc’.I’m. tr4dt›ction by G.J. Toonicr. II l. ?. p. T47 (I) H2*H. I.]] -I2I

Apogee



Perigee

Fig 2.18

We must now show that this situation corresponds to that of maximum anomaly, i.e. that the line of sight from the Earth to the celestial body, the straight line DHG, is then tangent to the epicycle.

Rolémée proves this by geometric reasoning based on the idea that the angular velocity of the star on the epicycle is equal to that of the epicycle on the deferent. Thus, i . \: i is equal to the angle traveled by the center of the epicycle from the point where it was when the star was at its apogee—namely ∠EDF s in Fig. 2 18. *As* this clearly shows, the difference between this angle (∠EAH = ∠EDF) et ∠ADH (representing *the anomaly)* corresponds to the angular distance separating the apogee of the celestial body from its orbit, as seen from Earth (. Or, d'une part on Figure 2.18) we have placed ourselves in the case where this is at 90°; and, on the other hand, this difference ∠AHD 1, is equal to i [66] Therefore, is a right angle. And the line DHG is indeed tangent to the epicycle.

65 Indeed:
DAH * AHD * ÜADH= l8‹i'
DAH+ YEAH - 181)"
=r AHD+ ÜADH= YEAH
ÙAHD - YEAH - ADH

Ptolemy then demonstrates that, according to the epicyclic hypothesis, when the directions of rotation are as he has chosen them (the epicycle rotating from west to east on the deferent and the star from east to west on the epicycle), the star takes longer to accelerate from its minimum speed (at apogee) to its average speed than it does to go from its average speed to its maximum speed (at perigee)." As in the case of J'excentrique and like Theon, Ptolemy considers it self-evident that the average speed corresponds to the case where the star appears to be at a quadrant of its apogee. He certainly starts from the principle that he has just proved by showing that in this situation, the line joining the tetre to the asfre is tangent to the epicycle. Indeed, it seems obvious that the direction of the sun's movement on the epicycle is in the direction of this line of sight, and this movement neither adds nor subtracts anything from that of the epicycle on the deferent; the angular velocity of the celestial body as seen from Earth is therefore equal to that of the epicycle, i.e., to the average velocity.

To compare the time taken by a star to go from its minimum to average speed on the one hand, and from average to maximum speed on the other, Ptolemy completes Fig. 2.17 by adding what we have indicated in dark red. Note that the arc from the apogee to H is greater than 90º by the arc KH, while the arc from H to the perigee is less than 90º by the
l' same arc. The first arc (ECiH) is therefore greater than the second (HZ) by twice KH; however, this arc is equivalent to the arc AG, which corresponds to the anomaly, AKAH being equal to JADG.[67]

Still within the framework of his preliminary theorems, Ptolemy demonstrates the equality of the anomaly equations for the same distance from the apogee and perigee from the point of view of the epicyclic hypothesis, comme il l'a as for the eccentric hypothesis. [6] He bases this on the figure below. [6] '

66 Ptolemy, f *'Almagest,* translation by G.J. Toomer, III. 3. pp. f J7- 148 [fi224, 1, 1 l - H235, 1.6].
67 *by Luck mr, Li s FHémeiits. ¥1).*
68 Ptoléméc. *f'Almagesie,* translated by G.J. Tootner, III, 3, pp.152- l fi3 {H230, 1. 14 - H232, 1.2J.
fi9 Piolemée, f ñfmepccr«, translated by G.J. Tourner. HI, 3, p.152, Fi*.3.8.

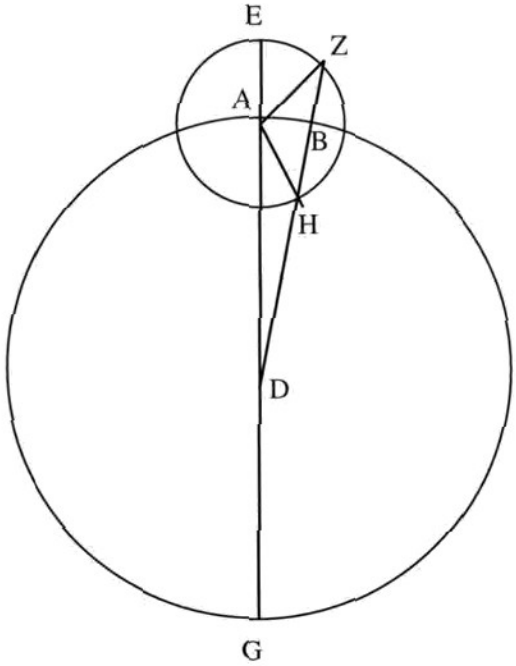


Fig. 2.19

The circle ABG represents the deferent and EZH the epicycle. Ptolemy draws the line DHBZ to represent the anomaly ADB corresponding to the positions H and Z of the star on the epicycle. The aim is therefore to show that these positions of the sun on the epicycle correspond to equal apparent distances from the perigee and apogee of the orb: that, in Fig. 2.20, which Ptolemy does not give, t D Z = x nDH. D notes that these angles are respectively equal to DZA and ZZHA (that is, in our figure, £IDZ= DZA and AfIDH= ZZHA). Now DZA and ZZHA are equal, since they constitute the angles on either side of the base of the isosceles triangle AHZ."

70 This triangle is isosceles because its sides AH and AZ *are* equal, forming the radii of the epicycle.

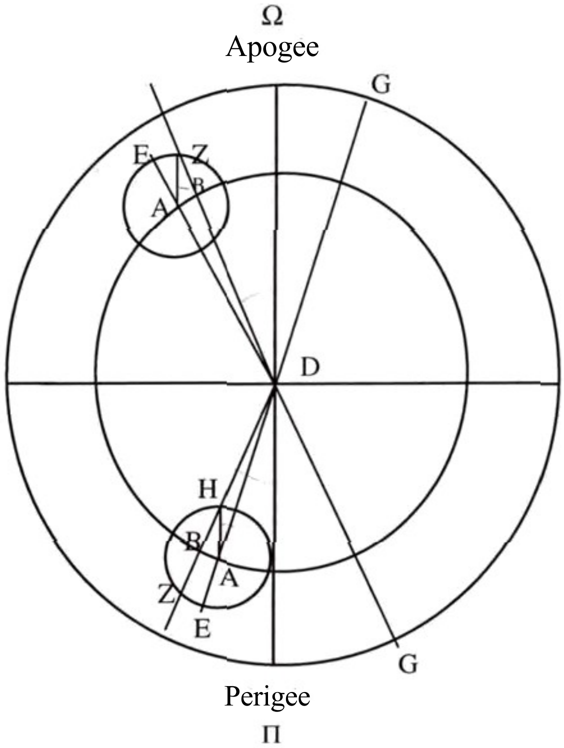


Fig.2.20

This demonstration concludes what Ptolemy considers to be "preliminary theorems,"(71) but we still need to consider the demonstrations that he and Theon propose to establish the equivalence of the eccentric and epicyclic hypotheses.

IV - **Demonstration of the equivalence of** the **two** hypotheses **A - In Theon**

A - In Theon

After demonstrating that the eccentric and epicyclic hypotheses can both account for the phenomena, Theon and Ptolemy demonstrate that they are mathematically equivalent.

71 Ptolemy, ***Almagest,*** translated by G.J. Toomer, III, 4, p.1 53 [H232, 1.2].

Theon approaches the problem in terms that may come as a surprise. Indeed, he proposes to demonstrate first, like Adrastus, that "the eccentric hypothesis is a consequence of that of the epicycle" [7°].11 He then goes on to demonstrate that 'the epicycle hypothesis is also a consequence of the eccentric hypothesis' (7[)], a demonstration that appears to be his own.

He actually offers two proofs of the first point. [74]

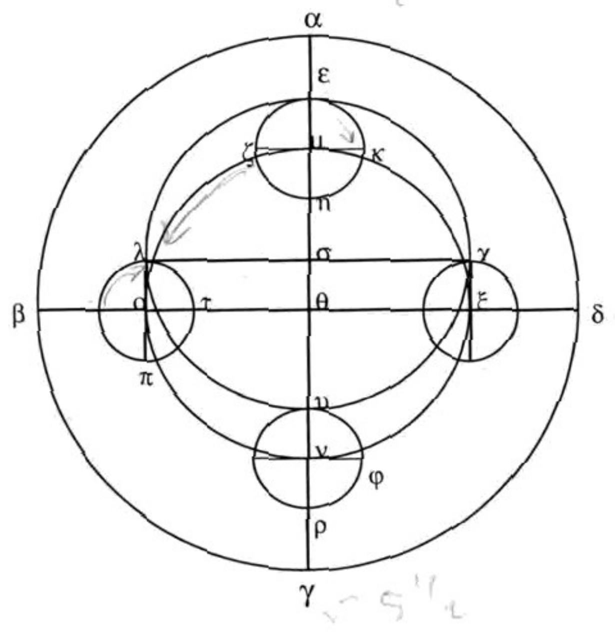


Fig. 2.21

In the first, [7]he considers a "deferent" circle qov§, and four positions of the epicycle, represented in the figure above, which he himself describes (o§@ representing the zodiac). He then defines an eccentric circle with a radius equal to the 'deferent' and whose center ‹f is located on the line joining ri to y, which designate II 5'/ and

72 Theon of Smyrna, *Exposition,* translation *by J.* Dupuis, p. 269, 1.14-15. "How the hypothesis of the eccentric follows that of the epicycle by consequence." Theon of Smyrna, *Exposition,* translation by J. Oelattrc, p. 162 [166, 1.1 I- 12|).

73 Theon of Smyrna, *Mposilion,* translation by J. Dupuis, p.269, 1.16- 17; ('this is also the case for the hypothesis according to the epicycle in relation to that according *to the eccentric', Theon* of Smyrna, *Exposition,* translation by J. Delaltre, p.162 [166, 1.12-J 3]).

74 Theon *of* Smyrna, *Exposition,* translated by J. Dupuis, p. 271, J. –p. 277, II l; Theon of Smyrna, *Exposition,* translated by J. Delattte, pp.162- 166 [156, 1.15 - 171, 1.2 J.

75 Theon of Smyrna, *Exposition,* translated by J. Dupuis, p.271, 1. l - p.277, J.21; Theon of Smyrna, *Exposition,* translation by J. Delattte, pp.152- 1fi4 [166, 1.15 - 169, 1.8].

1'/, — the apogee and perigee of the solar orbit. o is therefore located on the line of the apsides. Furthermore, this point is defined by construction as being at a distance d from the center of the deferent equal to the radius of the epicycle. Theon shows that the segments *az,* ou, and oy are equal to each other and equal to o8 and 8§, the radii of the deferent. These segments delimit quadrants of this eccentric circle because they form two perpendicular lines. Indeed, Z/ is parallel to o§ (because where and §/ are equal and form equal angles (in this case, right angles) with o(). And o§ is perpendicular to tiv.

Having defined the eccentric, Theon wants to show that the sun describes it when it moves on its epicycle. itself rotating on JioV(. Theon explains that when the epicycle moves from p to o, the sun moves from e to x on the epicycle, thus ending up at Z. He asserts that in doing so, the star will have "traveled a quarter of the eccentric," which must obviously be interpreted to mean that at all its intermediate positions between points E and k, the sun is also on the eccentric. Theon says the same for each of the other three quarters, with the sun moving from Z to u, from a 1 / and from \ to E, while the epicycle moves respectively from o to v, from v to § and from Ç back to (Each time, Theon asserts that when passing from one of the points k, u, / and e to the next, the sun is 'consequently' [7]on the eccentric in the intermediate positions. However, he offers no proof of this assertion, which he seems to consider self-evident. As Dupuis points out,[7] 'with this first demonstration, Theon only proves that the sun, when it describes the epicycle, is also on the eccentric at its apogee, perigee, and at points Z and \ — that is, at the moments that mark the middle of the time between the apogee and the perigee.

Theon then presents a second demonstration aimed at showing that the sun, when describing the epicycle, also describes the eccentric. He bases this on the figure below:

76 Theon of Smyrna, *Uposition,* translation by J. Dupuis, p. 273, 1.23.
77 Theon of Smyrna, *Uposition,* translation by J. Dclattrc, p.164 [168, 1.22 - 169, 1.2],
78 J. Dupuis, translation of *The Exposition* of Theon of Smyrna, Note p.27 i.
79 Theon of Smyrna. *Mposilian,* translated by J. Dupuis. p.27'î, 1.3-p,277, 1.21; Theon of Smyrna, *Exposition,* translation by J. Dclattrc, pp.l65-1 fi6 Î169, 1.9 - 171, 1.2]

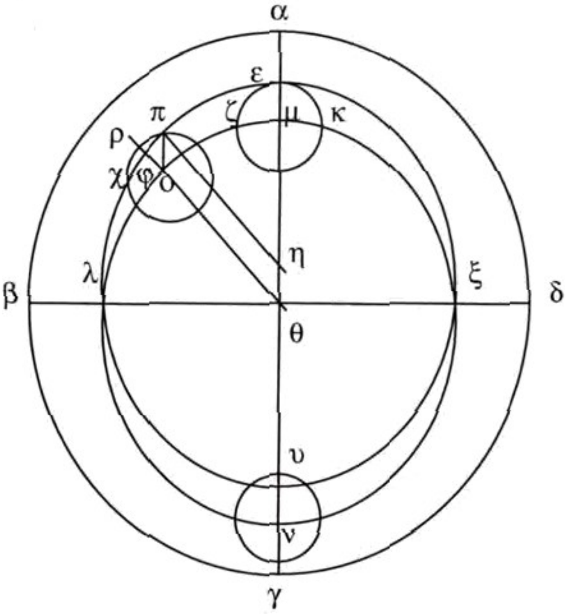


Fig. 2.22

Here he does not begin by defining the eccentric that the sun must follow. He simply indicates that he wishes to demonstrate that the star, as it travels along the epicycle, consequently describes an eccentric whose center he does not define in advance. Furthermore, he chooses to reason from any position of the epicycle on the deferent, which will allow him to prove what he wants: that the sun is *always* on an eccentric, et non pas only at quatre positions mentioned previously.

His demonstration is as follows. As the sun moves on the epicycle at the same angular velocity as the epicycle rotates, the angles pou and o8{i are equal. Theon takes point rj, located on the line of the apsides œy at a distance of 8 equal to the radius of the epicycle, pe. This point 9 will be the center of the eccentric, but Theon does not define it as such. He then notes that, due to the equality of angles pon and o8p, 8rj is

8tl Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.274; Theon of Smyrna, *Exf›osilion.* translation by J. Dclattre, p. t65.

81 Theon of Smyrna, *Exposftian,* translated by J. Dupuis, p. 275, 1.13; Thëon of Smyrnc, £.ryonffHf2. translated by J. Dclattrc, p. 165 [170, 1.4].

parallel to on. Furthermore, Bre and on are equal because on is also a radius of the epicycle. Therefore, rJn is equal to g2- 8o, since mj8o forms a parallelogram. Since 8o is equal to de, lpi is equal to ne, which means that, regardless of the point ri where the sun finds itself when describing the epicycle, this same point *is located* on *a* circle with center p and radius tje—therefore with radius 8p, the radius of the "deferent" since ne= eti+ pr|, p8= fry+ rJ8 and eu = 98. This time, Theon has clearly demonstrated that a celestial body describes an eccentric by describing an epicycle. He has also shown that the radius of this eccentric is the same length as that of the 'deferent', and that the eccentricity (the distance between the Earth and the center of the eccentric) is equal to the radius of the epicycle.

Theon further shows that for any position of the sun, the angle separating the star from its apogee as seen from the center of the eccentric, Aerjn, is equal to the angle it has traveled on its epicycle since that apogee, too. This is due to the fact already mentioned above that the angles pom and o8p are equal. Indeed, üo8;i is also equal to AETjn since is parallel to o8. Therefore, üxop is equal to Jcr}a.

Theon then sets out to demonstrate the opposite of what he was trying to prove in these two demonstrations: he now wants to show that the sun describes the epicycle by traveling along an eccentric. He returns to Fig. 2.22, but this time he begins by defining the eccentric ekB§, specifying that point e constitutes the apogee, at II 'i'/t, and that the center r is located on the line œ8, and therefore on the line of the apsides. He then defines two circles, one of which he will demonstrate plays the role of deferent, while the other is described by the sun in its movement on the eccentric, thus constituting an epicycle. He defines the first as the circle equal to the eccentric and with center 8 — ttov§ — and the second as the circle with center p tangent to the eccentric at c — eÇx.

Based on these premises, Theon draws the following conclusion. As in the previous problem, he deals with the general case: the sun is located at any point ri

82 qz is also parallel to iï 8o, but Thson, although mentioning this fact, does not need it.
83 Theon of Smyrna, *E.xposi1ion,* translation by J. Oupuis, p.277, l.22-p.279, 1.2.3; Theon of Smyrna, *ExPO$é1ion* translation by J. Delattre. pp.166- 167 [171, 1.3 - i 72, 1. t 6].

of the eccentric. He defines the circle whose radius is equal to 1 8i), the eccentricity, and whose *center* o is located both on the line parallel to rq and on the circle that will play the role of deferent (pov§). Theon must now show that ri is on this circle.

The segments irij and o8 are parallel by construction, and equal because they are radii of the circles eXv§ and ttov§, respectively. Therefore, mj8o is a parallelogram. And the segments xo and rj8 are also equal. Now r{8 is equal to po, so no is equal to po." This equality of no and po means that the point ri, wherever it is on the eccentric followed by the sun, is also located on a circle whose center is on the circle povf;, and whose radius is equal to the eccentricity, and therefore to the radius it has defined for the epicycle, eux.

However, Theon does not stop there, and sets out to demonstrate that the sun will be at point ri according to the epicyclic hypothesis or *at the same moment* that it is there according to the eccentric hypothesis. To do this, he must first show that the circle tip2 represents the position of the epicycle at the same moment *that* the sun is at point x of its *eccentric. He* must therefore prove that, at the same time that the sun travels along the arc en of the eccentric, the center p of the epicycle describing arc No, carries the epicycle to np/"(6) Therefore, arcs en and Jio are equal. This is easy for him because, or and 8q being parallel, as he has already shown above, angles er}n and }i8o are equal. Secondly, Theon must demonstrate that the sun moves on the epicycle from e to z in the same amount of time that the sun moves from e to ri in the eccentric hypothesis, and the epicycle, according to the other hypothesis, from p to o. Since the origin of the epicycle is diametrically opposite the earth in the epicyclic hypothesis in question (7), point t becomes point p when the epicycle is at o. Theon must therefore prove that angle er{a is equal to angle pou, which he does by demonstrating the equality of

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R4 Thdnn will only use the fact that *ÏlS are parallel later on.

85 Theon brings in the fact that r}8 is equal to pu ct, according to Theon *of* Smyrna, *exposition,* translation by J. Hctattre (p.167 [171, 1.19]) (Dupuis does not specify this; the passage in question is on p. 279, 1.5-6) that pu is equal to po er no to establish the equality of po and co, since it is the radius of *the circle* cÇr that he defined as being the epicycle.

8b Wïtëon äe Smyrnc, *MpasjtÎDn,* translation by J. Dupuis, p. 279, 1.15- 17; see also Théon dc Smyrnc. *Myosilinn,* translation by J. Delattre, p.167 [172, t.7-9).

87 The same applies to the model of the movement. t lunaire proposé par Ptolémée.

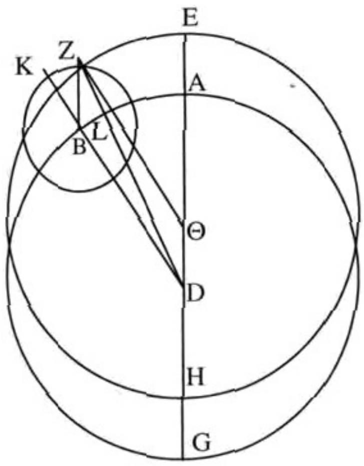
88 He reasons in fact from the arcs Ett and pu.

angles eqn and p8o. because he has already shown that }i8o is equal to pou prouvant le in parallel with no and rj8.

Theon thus demonstrated that the sun, as it travels along its eccentric, simultaneously travels along the epicycle of equal radius to the eccentricity, whose center moves on the concentric circle of the same radius as the eccentricity.

B - In Ptolemy

Ptolemy first addresses the question of the equivalence of the eccentric and epicyclic hypotheses *in* the case where the deferent and the eccentric are of the same size and the radius of the epicycle is equal to the eccentricity. He presents only one demonstration of this,[89] which aims to prove both senses of equivalence at once, rather than showing separately first that the eccentric hypothesis follows from the epicyclic and then that the reverse is true, as Theon does. Ptolemy therefore defines from the outset both the concentric circle ABG, the eccentric EZH of the same radius, and an epicycle located at any center B on **ABG,** and of radius equal to the eccentricity. Apart from the fact that he thus merges the two meanings of equivalence, his demonstration is very similar to Theon's second and third demonstrations.



[90]

Fig.2.23

89 Ptolemy, *Almagest,* translation by CI.J. Toomer, III, 3, pp.148-150, H225, 1.7 - H227, 1.10.
90 Ptuleine, *l'Almagest,* translation by €i.J. Toomer, III, 3, p. 149, Fig.3.5.

Rolémée sets out to demonstrate that the star will be at point Z, the intersection between the eccentric and the epicycle, at the same time according to both hypotheses. Like Theon, he does this by demonstrating that the 'arcs' EZ, AB and KZ are equal. However, he does so much more succinctly than Theon. He shows that BDO-Z is a parallelogram, which he does by indicating that its sides are equal in pairs (ZO- ä BD and BZ ä DÖ). 11 He deduces that the angles EO-Z, ADB, and Z8X are equal, and that therefore the arcs they subtend are sinùJaizcs.

Ptolemy emphasizes more than Theon that the apparent positions will be the same in both cases, which Théon leaves implicit; in particular, he takes the trouble to define the point on the arc AL. apparent distance between the sun and the apogee of its orb. He treats AL as an arc of the ecliptic because, although points A and L are located on the deferent, the latter is concentric. The arc has the same angle as a similar arc on the ecliptic. He also emphasizes that the anomaly equation is the same regardless of the hypothesis·which he demonstrates by pointing out that it is equal to the angle DZO- in the eccentric hypothesis and to the angle BDZ in the epicycle hypothesis. However, these two angles are equal since they are formed by the intersection of the line DZ with the two parallel lines BD and ÖZ·

Unlike Théon, Rolémée also demonstrates that the two hypotheses can be equivalent without the deferent and the eccentric being the same size and the radius of the epicycle being equal to the eccentricity, provided that the ratio of the eccentricity to the radius of the eccentric in the eccentric hypothesis is equal to the ratio of the radius of 1epicycle to that of the deferent in the epicyclic hypothesis. [5] To do this, he first defines the position of the star according to the latter. He then demonstrates that it will be at the same angular distance from its apogee position in several cases of

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91 Ptolemy, *Almagest,* translation by G.J. Toomer, III, 3, p.149, H22G, 1, l -6 & H226, 1,14-1 5.
92 Ptolemy, *Almagest,* translation by C¡.J. Toomer, III, 3, p.149, H226, l.15-H227, 1.1 .
93 Ptolemy, J *'Almagesle,* translated by G.J. Toonier, 111, 3, p.149, H226, 1.4-5 & H227, 1, l -2.
94 Ptolemy, *Almagest,* translated by G.J. Toomcr, III, 3, p.149, H227, 1.2-5.
95 Ptolemy, *Almagest,* translated by G.J. Toonier, III, 3, pp.150- l51, H227,l.l l -H229, 1.1 3,

different figures according to the eccentric hypothesis, if the above condition is met. In this respect, contrary to his previous demonstration, it would seem that Theon's approach falls back on the position defined by a hypothesis to then show that the situation is the same under the other hypothesis. However, Ptolemy's approach remains different here on one fundamental point: for him, it is not a question of proving that the sun, by describing the epicycle, also describes the eccentric, but simply that under the conditions he has set, the apogee-earth-star angle, ADZ, will be the same.

Ptolemy therefore represents the star in position Z on the epicycle EZ, which is located at any point B on the deferent (Fig. 24), [6] and draws two eccentric arcs, one larger (centered at K) and the other smaller (centered at N) than the deferent.[7] D considers the positions Ö and M of the sun on these circles, for which the star will be seen from the earth in the same direction as in Z — i.e. such that ADZ = ADO
= ADM — that is, the points of intersection of the line DZ with these circles. Ptolemy's demonstration therefore consists in proving that the star travels the arcs HO and LM in the same time as the epicycle travels the arc AB and the star travels the arc EZ (the latter two durations having already been shown to be equal).

He notes that, based on his hypothesis, two of the sides of each of the triangles **BDZ,** MDN, and DKO- have the same ratio: BZ/BD= DN/MN= DK KO. Furthermore, if we consider the two angles that are not formed by these sides for each of these triangles, we see that one of these two angles is equal for all three triangles. Indeed, MDN and **DKÖ** share angle **ADZ,** which is equal to angle BZD—because DA and BZ are parallel, as we saw earlier. Therefore, these triangles are equiangular. The angles BDZ, DMN, and DOK are therefore equal. The fact that BDZ is equal to DMN implies that BD is parallel to MN, and the fact that BDZ is equal to DO-K implies that BD is parallel to OK. The parallelism of these three lines implies that

96 Ptolemy, *Almagest,* translation by G.J. Toomer, III, 3, p.l5ô, Fig.3.6.
97 The centers must be on the right side of the apsides, not only so that the angular distance of the star from its apogee can be easily calculated from the figure, but above all because the apogee must be in the same place regardless of the parameters used.

The angles AT)R. AU k4 and AK(-) are equal, and therefore equal to the angles CH, 1 X f el Hai.

sont similaires.

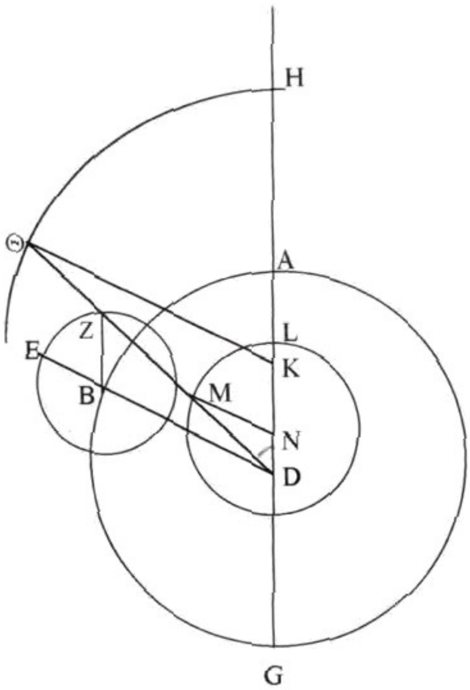

Fig.2.24

I.e. the fact that Ptolemy took the trouble to demonstrate the equivalence of the two hypotheses by basing it on the equality of the apogee-Earth-star angles, and not on the notion of 'description' of the same curve — and we can assume that this is perhaps what contrary is usedpour quoi, concept in the previous demonstrations either pas is very interesting from an epistemological point of view. We can say that in this respect Ptolemy places himself more than Theon in an approach that is not only more abstract, but also more rigorous, or "scientific" as we would say today, consisting *in trying* to "save appearances." Indeed. In the case we have just seen, the star is not physically in the same place in either hypothesis, but simply in the same direction as seen from Earth. It is therefore advisable to remain cautious, because the fact that he occasionally uses such a

This approach does not necessarily imply a more generally "positivist" perception on the part of Ptolemy, as we shall see in Chapter 6.

CHAPTER III: THE MOTION OF THE PLANETS

The epicycle and eccentric hypotheses we studied in the previous chapter provided a satisfactory explanation of the movement of the sun in longitude. However, especially in the simplest forms we described, they were inadequate for the movements of the other planets. Ptolemy does not hesitate to say of the task of explaining the latter in terms of uniform circular motions that "... on many grounds, we must think that it is difficult, and that there is good reason why no one before us has yet succeeded in it." Despite its non-technical nature, Theon of Smyrna's work is one of the best sources of information on the unsatisfactory models to which Ptolemy refers, as it was probably written shortly before *the Almagest.* This chapter attempts to determine what these models were and what Ptolemy's contribution consisted of.

1 - The order of the stars

Like Theon and Ptolemy, before addressing the question of geometric models themselves, we will deal with a preliminary problem: that of the order of the stars in relation to the Earth. Clearly, this central question was not unanimously agreed upon. Theon presents different opinions, and it does not seem to be *merely* a historical reference for him; it seems more likely that they were widely accepted.

1 PtoJéméc, *t'Almoyeste,* translation by G, J. Toomer, lX.2, p.420, H208, 1.9- 1 1.

of some consideration in his time. Theon himself certainly has a preference in this matter, but he does not hide the fact that there are alternatives. As for Ptolemy, although he gives a more definitive opinion on the matter in both *the Almagest* and *the Book of Hypotheses,* he also admits that this is only an opinion and that there is uncertainty on the subject.

Despite these uncertainties, it is clear that there was agreement on a number of points. This is what Ptolemy asserts in *the Almagest,* referring to important astronomers.[2] There is thus unanimity on the fact that the closest celestial body to the Earth is the Moon, and that the most distant are, from closest to furthest: Mars, Jupiter, Saturn, and, beyond them, the sphere of the fixed stars. In this respect, the cases considered by Theon confirm *what Ptolemy* tells *us, since they all* seem to have these characteristics—although Theon is not very explicit about Eratosthenes' system.

The only problem, therefore, is the positions of Mercury, Venus, and the sun: there is certainly agreement that they are between the moon and Mars, but not on the positions they occupy in relation to each other. Indeed, Theon attributes to "certain Pythagoreans" the opinion that the stars are arranged in the following order, from closest to furthest from the Earth:

moon, Mercury, Venus, sun, Mars, Jupiter, Saturn, fixed stars (ri)

Such, at least, is the opinion of Alexander of Aetolia, whom he quotes, and Theon mentions no other author. Theon attributes this preference for the order [ce] to an analogy: the

2 Ptolemy, f *'Atmage.sfe,* IX. I, translation by CE./. Toomer, p.4 J9.

3 in ascending order of distance from the earth; in what follows, we will always present the hierarchy of the stars in this order.

4 According to Thomas Heath, this is in fact AJe xandrc *d'Cphüse,* a contemporary of Cicero (T.Healh, *Arislarchus of Samus, the Ancients Copet nicus* p.l 12).

5 This system is obviously very different from that adopted by other Pythagoreans (in particular by Philolaos, who may even have been its author, if we are to believe Astitis), for whom the *earth* revolves around a central fire, hidden by a counter-earth (T. Heath, *Arixiarchus u]' Samos, the* Ancients *Copernicus,* pp.94-95). Furthermore, Theon disagrees with Alexander because the latter attributes sound to the earth as to the other stars due to the movements of their spheres, which Theon criticizes as impossible due to the immobility of the earth (Theon dc

The sun must stand in the middle of the planets "as the heart of the universe and the most capable of command." [6]This reflection suggests that he thinks it is the position of the sun that poses a problem, since that is the one he takes the trouble to justify.

The way in which he presents Eratosthenes' views seems at first glance to confirm that this is indeed his perception of the problem. Indeed, Theon does not even indicate the order of the stars according to his famous predecessor, but only the place that the latter assigns to the sun: just above the moon. One might therefore be tempted to assume that Eratosthenes attributed the following order to the celestial bodies:

moon, sun, Mercury, Venus, Mars, Jupiter, Saturn, fixed stars [§]

This interpretation seems, however, to contradict what Theon goes on to say. He sets out the two hierarchies accepted by "mathematicians" and points out that they differ from those he has just described; one of them corresponds precisely to the interpretation we have just given of Eratosthenes' views. The opinion that Theon attributes to the *latter is* therefore unclear.

On the other hand, he is sufficiently explicit about the views of those he calls "mathematicians" that there is no ambiguity. They are therefore divided between two opinions: on the one hand, as we have just said:

Juno, the sun, Mercury, Venus, Mars, Jupiter, Saturn, *and the* fixed [γ]

stars; and, on the other hand:

moon, sun, Venus, Mercury, Mars, *Jupiter,* Saturn, fixed stars[7] [δ]

Smyrna, *Exhibition* translation by F. Dupuis, p. 231, 1.15-17.) 11 However, we should not conclude that Alexander believed the earth to be in motion, as his description of the order of the spheres as reported by Theon seems to have been purely geographical.

6 Theon of Smyrna, *Exposiiian,* translation by J. Dupuis, p.227. Theon will take up and develop this analogy further (pp.301-305), as we shall see.

7 The passage from Plato's *Republic* that Theon then quotes illustrates this: Plato arranges the stars according to this second order.

According to Theon, "mathematicians *therefore* agree *on* the position of the sun, beyond the moon but below the planets. What poses a problem for them are the positions of Mercury and Venus in relation to each other."(8)

Theon presents a highly original alternative, which would resolve this problem by rendering the distinction between (y] and (ô] meaningless. Indeed, in his exposition of the physical models of the planets, he suggests that Mercury and Venus may revolve around the sun. He does not categorically endorse this possibility, but it is clear that he prefers it.° We will return to the advantages of this proposal in more detail after discussing the geometric models of the stars when we address the question of physical models in Chapter 5.

This idea of Venus and Mercury revolving around the sun is nowhere to be found in what has come down to us from Ptolemy's work.

Furthermore, even the configurations previously mentioned by Theon do not always correspond to what Ptolemy tells us. Of course, we find the moon closest to the earth, and Mars, Jupiter, Saturn, then the fixed stars furthest away. Nevertheless, in the *Book of Hyyothè.ses,* he states that there is no doubt that Mercury is closer to the earth than Venus. We thus have:

moon, Mercury, Venus, Mars, Jupiter, Saturn, fixed stars [ε]

Ptolemy thus **reduces** the problem of the order of the stars to that of the position of the solar orb in relation to the others: between the moon and Mercury or between Venus and Mars.

g Theon of Smyrna, frposfrfoa, *translation by* J. Dupuis, p.237.
9 Theon of Smyrna, *Exposition,* translation by J. Dupuis, pp. 301-303. This does not mean, however, that [y] ct (fi] are in fact the same. Indeed, Theon specifies that there is disagreement among mathematicians; these are therefore two different configurations.
10 Thèon dii referring to this: 'It will be understood that this position and order are all the more true 9 ue...', Theon of *Smyrna, Exposition,* translation by J. Dupuis, p.303, 1.4-fi; see also Theon of Smyrna, Exposition, translation by J. Dupuis, p.313, 1.12-13.
11 Ptolemy, *Almagest,* IX.1, translation by G.J. Toomer, p.419.
12 Ptolëmëc, le fine 'êtes' Ny ùe.res, translation from Arabic by R. More1t›n, in *MIDEO,* 21 (1993), p.ô2.

Although he does not explicitly state this in / *'Almagesle,* it seems clear that this was already his opinion when he wrote this work. Indeed, he already emphasizes the problem of whether the orb of the sun is closer or farther than those of Venus and Mercury. When he refers to the uncertainty regarding the position of these two stars, he treats them together. Furthermore, after listing Saturn, Jupiter, and Mars in order of distance from the Earth, he first refers to Venus, as if this celestial body were farther away than Mercury. It therefore seems that, even in *the Almagest,* Ptolemy believed that there were two possible configurations:

moon, Mercury, Venus, sun, Mars, Jupiter, Saturn, fixed stars [Ç]= [a]

moon, sun, Mercury, Venus, Mars, Jupiter, Saturn, fixed stars [Ç] [rJ]= [§]

Ptolemy further specifies that the first [Ç] was unanimously accepted by the most ancient astronomers—which is rather consistent with Theon's reference to "certain Pythagoreans" for this configuration—and that the second had only been considered more recently, and by some only. According to Toorner. Plato, Eratosthenes, and Archimedes were certainly among these more recent astronomers. Theon does not mention Archimedes, and what little he says about Eratosthenes' views is indeed consistent with the order [qJ. However, what he reports about Plato does not correspond to this.

So far, we have focused on the nature of the *ARn* configurations proposed to do justice to the complexity of this problem, but we must of course also examine the reasons why they were adopted. Theon, for his part, is not very explicit about the motivations of the authors to whom he refers. He describes and criticizes the implications of what Alexander

13 Ptolemy, *Almagest,* IX. 1, translation by G.J. Toomer, p.419.

of Aetolia and Eratosthenes for musical harmony due to the movements of the spheres. One might therefore be tempted to think that these musical considerations play a role in determining the order of the planets, but in fact this does not seem to be the case.

However, the reasons why Theon prefers the hypothesis of a movement of Venus and Mercury around the sun are more obvious. They are partly based on considerations that could be described as allegorical. Theon draws a parallel between the universe and the animal body, explaining that, in the same way that the center of the "animate creature" [4]as the seat of consciousness, which he identifies with the heart, differs from the center of an animal's body—in the sense of the center of its volume— we can speak of a center of the universe that is different from the middle of the universe (which is the earth): the sun. The metaphor seems to be based in part on the fact that the sun is hot and that this quality is associated with a certain notion of dynamism, one might even be tempted to say energy, certainly attributed to the heart because of its perpetual beating. Linked to this symbolism is the notion that the heart is the seat of consciousness — 'cause [...J of all movement from one place to another. source of our desires, our imagination, and our intelligence' — and that according to a myth, 'the soul of the world was born' in the sun.[(6)]

But Theon also puts forward observational arguments. Indeed, when he describes the phenomenon of occultations, he specifies that Venus and Mercury "even appear to eclipse each other, depending on whether one of the two planets is higher than the other, due to their sizes, obliquity, and the position of their circles." [(f7)] This implies, of course, that each of them is alternately closer to and further from the Earth, and only one of the configurations he proposes, in which the two stars revolve around the Sun, is compatible with this. Once again, Theon is not categorical, but he believes that this is what has actually been observed."

14 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p. 303.
15 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.303, 1. 1f›-18.
16 Theon of Smyrna, *Exposition,* translated by J. Dupuis, p. 303, 1.2f›.
ï 7 Theon of *Smyrna, E:rpoyjlion, translated by J.* Dupuis, *y.? 1?.*
1 8 **He specifies**: '[the] fact is **not** easy to observe...', implying that it is nevertheless possible, and he explains what the difficulty consists of — namely, the limited visibility of Mercury and Venus. Theon of Sniyrnc, *Expo.sitinn,* translation by J. Dupuis, p.313, 1.1 2-1ô.

However, it is not clear whether the underlying reason for this is his confidence in the observations concerned rather than an aesthetic preference for the configuration in question , or linked to the allegory we have just outlined.

Ptolemy, meanwhile, in *his Book of Hypotheses,* explicitly attributes order [e] to the observation of occultations: stars whose orbits are closer to Earth sometimes obscure others. Theon does not explicitly refer to occultations as a means of determining the order of stars when discussing them. However, when he goes on to explain the phenomenon of occultations, as we have seen, it is clear that he *considers it* to offer this possibility·According to both authors, this type of observation is the reason for their disagreement regarding the relative positions of the ortes of Venus and Mercury. Clearly, Ptolemy only admits the occurrence of occultations of Venus by Mercury, while, as we have said, Theon believes that Mercury can also occult Venus. Ptolemy only admits the occurrence of occultations of Venus by Mercury,([2]) whereas, as we have said, Theon believes that Mercury can also occult Venus.([22])

Furthermore, Ptolemy attributes the uncertainty regarding the position of the solar orb relative to those of Venus and Mercury to the fact that it cannot be determined from occultations. In fact, according to him, the reason why some more recent astronomers have preferred hypothesis [rJ], according to which the solar orb is below those of Mercury and Venus, is because no obscuration of the sun by these two planets has ever been observed.[4] li considers this argument to be invalid for two reasons. **On** the one hand, *even* if the orbs of Venus and Mercury are located below the solar orb, for there to be an obscuration, these celestial bodies must have the same position as the sun in latitude when they are at the same longitude. However, according to Ptolemy, this is very rare. "It is therefore not surprising that no obscuration is observed at

19 Ptolemy, ***The*** *Book of Hypotheses,* translated from Arabic by R. Mnrelon, in *MIDECt,* 21 (1993), **p.62.**
20 Tbëon dr Sinyme, Ar z'J/íon, translation by J. Dupuis, pp. 31-31.
21 Ptotéméc, The Book ***of Hypotheses,*** translated from Arabic by R. Morton, in ***MIDEO,*** 21 (1993), p.62.
22 Theon of Smyrna, *Exposition,* translated by J. Dupuis, pp. **31-313.**
23 Ptolemy, *Almagest,* lX.l, translated by G,J. Toonier, p.419.
Ptolemy, ***The Book of*** *Hypotheses,* translated from Arabic by R. Morelon, **in *MIDEO,* 21** (1993). pp.62 & 4.
24 Ptolemy, *Almagest,* IX.*1,* translation by G.J. Toomcr, p.419.
25 Why he does not consider *the* rotations of the five plates relative to each other to be more *complex* — even though **much** smaller than the sun — **is** unclear.

conjunctions, even though the orbits of Venus and Mercury are inferior to that of the sun. On the other hand, the sun is so much larger and brighter than the two planets that even if they did pass in front of the luminary, it would not be noticed."

Ptolemy mentions that another way of *determining* the order of *the osu:es is* to estimate their distance by measuring their parallax. However, the accuracy of observations at the time only allowed this to be done for the moon and, at a pinch, for the sun; for the five planets, it was naturally out of the question· (7)

I) There are therefore no satisfactory direct means of determining the position of the solar orb. In *the Book of Hypotheses,* Ptolemy even alludes to the possibility—albeit unlikely—that the orb of the sun may be greater than those of Mars, Jupiter, and Saturn, since there can be no certainty on the matter.[2] ' Since he must nevertheless make a choice, he decides to place it between that of Venus and that of Mars [Ç] — hence the expression 'lower planets' to designate Mercury and Venus, and 'upper planets' for Mars, Jupiter, and Saturn. The reasons he gives for this decision are different in *the Almagest* and in the *Book of Hypotheses.* In the former, his motivations appear largely aesthetic and personal. They stem from the fact that, in this way, the solar orb forms a separation between planets whose elongation can be any (Mars, Jupiter, and Saturn) and those whose elongation reaches a maximum (Venus and Mercury)." By contrast. in *the Book of* Hypotheses, already benefiting from the results he presented in *the Almagest,* he gives another reason, which seems less subjective, and which he presents as decisive: having established the ratios of the minimum and maximum distances of the different planets to the Earth's radius, he thus has a means of calculating these distances, and in

26 Ptolemy, *The Book of Hypotheses,* translated from Arabic by R. Morelon, in *MlDECt,* 21 (1991), p.62.
27 Ptolemy, *Almagest,* IX,1, translated by G.f. Toomer, *p.419,* Ptolùmëe. *The Book of Hypotheses,* translated from Arabic by R. Morelon, in *MlOECt,* 21 (1993), p.62.
25 Pttilémée, fe *Book of Hypotheses,* translation from Arabic by II. Morelon, in *MIDGH,* 21 (1993 , pp.62 & 64.
29 Ptolëmée. *l'Almoffesie,* lX. t , translation by G.J. Tourner, pp.419-420. As we have already mentioned, the Pythagoreans to whom Theon refers attribute the same order to the planetary orbs for reasons that we can also describe as aesthetic, although they are more related to an analogy between the sun and the heart.
30 see chapter •;.

some kind of way to verify the position of the solar orb. The result he obtained, although imperfect, was close enough to what he expected to confirm his hypothesis. Nevertheless, as Toomer points out, his results are at odds with the phenomena of Another point of view: according to the distances he calculates for Mercury, this planet should exhibit a significant parallax." Ptolemy does not mention this problem, and we can only speculate as to whether he was aware of it. Nevertheless, in *the Book* of *Hypotheses,* he gives yet another argument for his choice to place the sun's orb above Mercury and Venus. In this configuration, the planet with the most regular motion, i.e., Venus, is thus closer to the sun, the star with the most regular motion of all. Mercury is close to the moon and the place of air, and Ptolemy seems to think that this explains the particularly complex movements of these two planets, whose models share a unique feature.[33] Ptolemy further specifies that thus, [the sun] is in the middle from all points of view, which is reminiscent of Theon's allegory.

11 It should be noted that neither Theon nor Ptolemy propose classifying the orbs according to the speed of the planets.[5] However, since the celestial bodies on which there is agreement (the moon, Mars, Jupiter, and Saturn) have periods of increasing length with their distance, one might suppose that this is an argument in favor of configuration [Ç], which Ptolemy prefers. Why he does not use this argument is unclear.

31 G.1. Toomer, ***Ptolemy's** Almagest,* p.420, note 4. Ptolemy had, however, explicitly considered the requirement that this model should not lead to an observable parallax for Mercury and Venus. (Ptolemy, The *Almagest,* IX.1, translation by G.J. Toomer, p.420).
32 Ptolemy, ***Book of Hypotheses,*** translated from Arabic by R. Morelon, ifl ***MIDEO**,* 2 l (**1993), p.68.**
-33 Their deferent **is** animated by a ***royal*** motion***, or, as*** we shall **see.**
34 Ptolemy, *Book of Hypotheses,* translation from the Arabic by R. Morelon, in *MIDEO,* 21 (1993), p.68.
35 Theon seems to allude to this when he says of Plato that he "... shows the order of these spheres in relation to [...] the speed of their movement in the opposite direction to the universe." (Theon of Smyrna, *Exposition,* translation by I. Dupuis, p.23fi). However, he **is** simply referring to the fact that **Plato** indicates the relative speeds of the astms, which he designates by their order. (Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.2371.

II - The sphere of the fixed stars

Before focusing on the geometric models themselves, however, we must first clarify a few points about the sphere of fixed stars as conceived by the two astronomers, since it is in relation to this sphere that the movements of the planets are observed. As we saw in Chapter 1, Theon and Ptolemy attributed the diurnal motion of the heavens to the rotation of this sphere of fixed stars from east to west, in which they certainly agreed with the vast majority of astronomers of their time. They also agreed that the "fixed stars" effectively maintain their positions relative to each other. The difference between the two authors lies in the fact that Theon asserts this without justification, whereas Ptolemy, on the contrary, bases his assertion on data. He compares contemporary observations with those of Hipparchus, which the latter compiled in his famous catalog, the first of its kind to his knowledge. The way in which Ptolemy *presents* these comparisons in *the Almagest* is not particularly rigorous; he merely indicates alignments, rather than comparing the coordinates of the stars." 11 nevertheless made a very precise comparison:

> If one were to match the above alignments too against the diagrams forming the constellations on Hipparchus' celestial globe, he would find that the positions of the [relevant stars] on the globe resulting from the observations made at that time [i.e. by Hipparchus], according to what he recorded, are very nearly the same as at present [40].

Indeed, Ptolemy does have the coordinates determined by Hipparchus for a number of stars, and he knows the contemporary coordinates relative to the ecliptic.[4] ' If he compares his data with those of Hipparchus in terms of 'il donne —

3fi TMon de Smyme, *Exposition,* translation by J. Dupuis, p.221, 1.6-8; Théon dc Smyrne, *Exposition,* ~~translation by J. Delattrc, p.127, [13~~4, 1.1-3].
37 Pto]érnëe specifies that he himself made observations **(/"*Almagerle,* VII,** 2, **p.32d,** H 15, 1.9-1O). J8 Piolëntëe, / :ï/ ytuze, iraduriion by G. J. Tnomer, VII, 1, p.321, H3. 1.4 - p.327, H11, 1.21.
39 Piolémée, ***J'Almozeste,*** translation by G. J. Toonier, V1I, 1, p.322, H4, 1. h - p.327, H11, 1.21.
40 Ptolemy, *Almagest,* translated by G. J. Toomcr, VII, 1, p.327, H11, 1.22 - H12, 1.4. 4 1 Ptolèméc, /â/mnpez/e, Introduction by G. J. Toomcr, VII, 5 - VIII, 1, pp.341-399.

alignment, it is because the coordinates differ. However, this does not imply any movement of the stars relative to each other, as their coordinates between the two epochs varied in the same way for all of them. Ptolemy therefore attributed this variation to a slow movement of the sphere of fixed stars from west to east. Hipparchus himself had already noted this, based on earlier observations([42]) and he is credited with the discovery of this "precession of the equinoxes." Ptolemy calculates its speed at approximately 1° per century([43]),a value that also corresponds to that obtained by Hipparchus, according to a passage from his work entitled *"On the Length of the Year,"* which Ptolemy quotes. He then sets out to verify Hipparchus' conclusion that the rotational movement does indeed take place around de l'the ecliptic and not that of the equator: he verified that the latitudes of the stars—i.e., their distances from the ecliptic—are constant, while their declinations—i.e., their distances from the equator—are not.[46] 11 It follows that the variations must be due to a movement from west to east of the sphere of fixed stars, as he had announced.

[47] Theon does not mention the phenomenon. This could be because Hipparchus' conclusions on the matter were not considered reliable until Ptolemy's work confirmed them. It may also be due to the non-technical nature of Theon's work, who may have preferred to ignore such a weak movement, or to the fact that his work is intended as an l' introduction to Plato's work, who was not familiar with the phenomenon.

42 Ptolemy, / g/nayesre, translation by G. J. Toomer, VII, 1, p.321, H2, 1.17 - H3, 1.4.

43 ~~Ptolemy, *Almagest,* translated by~~ G. J. Toomer, VII. 2, p.327, H12, 1.17 - p.329, H16, 1.9: also *Ptolemy, Book of Hypotheses, VIDEO,* pp.20-22.

44 According to a passage from his work entitled *De la longueur de l'unnée (On the Length of the Year)*, which Ptolemy himself quotes (Ptolemy, *Almagest,* translated by G. J. Toomer, VII, 2, p.328, H15, 1.12 - H16, 1.2). Hipparchus had also developed his hypothesis of the precession of the equinoxes in a work entitled *On the Completion of the Solemn and Equinoxial Points (*Ptoléméc, *1'Alrnugestv,* translation by G. I. Toomer, V1 327 8)

45 Ptolemy, *Almagest,* translated by G. J. Toorner, VII, 2, p.32R, H15, 1.12 - H16, 1,2.

46 ~~Ptolemy, *Almagest,*~~ translation by G. J. Toomer, VII, 3, p.329, H1fi, 1.10 - p.330, H19, f.10.

47 One could also say that the east-west movement of the sphere of fixed stars is simply slower than suggested by diurnal phenomena, but Ptolemy prefers to express the phenomenon in terms of this second west-east movement.

III - Geometric models of the planets

The geometric models that Ptolemy developed in *the Almagest* to explain the movements of the different planets in longitude form the core of this work, and indeed of his entire astronomical oeuvre. Indeed, while the model he proposes for the sun is due to Hipparchus, as he himself indicates, those for Mars, Jupiter, Saturn, Venus, Mercury, and the moon were most certainly developed entirely by Ptolemy himself. For the sake of simplicity, we will not follow the order of *the Almagest.* We will present the models from the simplest to the most complex—that is, in the order we have just indicated—and compare them to those known to Theon. It should nevertheless be noted that such an exercise faces a major difficulty: Theon's exposition is incomparably less detailed than that of Ptolemy. Above all, he does not deal with each planet individually. At most, he indicates their average speeds. This is obviously due to the non-technical nature of his work. However, there seems to be a second, much more instructive reason for this brevity: according to Theon, it seems that the motion of all the planets was thought to follow essentially the same model. We will therefore present the latter first, before describing those put forward by Ptolemy, which can be grouped into three types: one for the outer planets and for Venus, although there are some slight differences between the latter and the other three; one for Mercury; and another for the moon. We will also discuss the model of the sun, the parameters of which we did not give in Chapter 2.

It should also be noted that, unlike Theon, Ptolemy wants to provide a means of determining the position of a celestial body at any given time, using tables. Generally speaking, these tables indicate average values and the corrections that need to be made to them due to various anomalies. Ptolemy then gives the appropriate algorithms for

4B 7hdon cÏc *Sm e, Eposiiiarr,* rmducrion dr *1.* Dupuis, p.221, /.g-27: Tkéon de *ñmyrna, Eçnsiiion,* tra4uctinn dc 1. Dclatirc, p.129 [I 3*î, I.14 - 136, I.8j.

obtain the positions of the planets and luminaries from these tables. A significant portion of *the Alinageste* thus consists of explaining the methods by which Ptolemy obtained these positions. Naturally, we cannot detail these demonstrations for all the stars. However, we will do so for the sun, as an example.

A - The movement of the planets according to Theon

As we have said, it seems that for Théon, the movement of planets in longitude can be represented by the same fundamental model—with slight variations in parameters, of course. Or rather, we should say by the same fundamental models, because what he describes is sufficiently close to the simple eccentric and epicyclic hypotheses [0]that he can still differentiate between them and treat them as alternatives. He therefore believes that the motion of all planets can be accounted for by modified versions of one or the other hypothesis.

In the case of the epicyclic model, Theon explicitly states that the deferent is concentric. However, the first difference from the simple hypothesis used for the sun is that the angular velocity ‹q of the planet on the epicycle is different from the velocity ri of the epicycle on the deferent, [z]For most celestial bodies, @ is greater than , except for the moon. 53

Secondly, it seems that the ratio of the radius of the epicycle to that of the deferent varies depending on the celestial body in question. In fact, Theon refers to the "inequality of the circles" due to which the

49 We will discuss variations in latitude in the next chapter. Suffice it to say here that, for Théon, the circles concerned are **inclined** relative to **the ecliptic** by an **angle** that depends on the other circle considered. He indicates **this explicitly** in the case of the eccentric, but it seems highly probable that he thinks the same for **the** other **hypothesis.**
50 i.e. **as** we explained **in the previous chapter.**
51 or of the epicycle around its **center.**
5* or **from** the deferent **around its** center. Theon **of Smyrna, *Exposition,*** translation by J. Dupuis, **p.281**, **1.29-33;** Theon of Smyrna, ***Exposition,*** translation by J. Delattre, p.] 70 [**173,** 1.21 **-23]; &** Theon of Smyrna, *Mposiiion,* translated by J. Dupuis, p.299, 1**.22-24; Theon of** Smyrna, ***Exposition,*** **translated by J. Delattre, pp.183-** 184 [**185, 1**.19-**186, 1.2J (in** fact**, this second passage refers to** a **physical** model **that** involves a significant subtlety compared to the way we express it here, **but** it is indeed **what Theon** describes in terms **of speeds measured *by*** the "***vitess*** " (the speed of light) relative to the " " (the speed of light in the vacuum).
53 Theon **of Smyrna, *Uposylon,* translation by J.** Dupuis, **p. 283, 1.1-12;** Theon of Smyrna, *Expositian,* **translation by J.** Oetattre, pp. 170-171 [f 74, *L4-l41.*

speeds differ from one planet to another, and we can see what he might be referring to if not this ratio."[4] In fact, this does not strictly speaking constitute a difference in model compared to that of the sun, but rather a difference in parameters. Indeed, this ratio corresponds to

l'excentricity6 in the eccentric model, and this *is* a value that we determine.

Thirdly, apart from the moon, the direction of rotation of *the planets on* the epicycle is the opposite of that in the simple epicyclic hypothesis corresponding to the movement of the sun: the five planets rotate from west to east, in the same direction as the epicycle on the deferent. We find this peculiarity in Ptolemy's models. In fact, it reflects the fact that the true position of these celestial bodies is ahead of their average movement from west to east —

i.e. further east —)when they move from their apogee towards their perigee; in other words, their equation of anomaly *is added to* their mean position.[5] '

In the case of the eccentric hypothesis, Theon indicates, on the one hand, that the eccentricity varies from one planet to another·This corresponds to the second remark we made for the epicyclic model, i.e., the ratios of the radii of the epicycle and the deferent vary depending on the star in question.

The real difference from the simple hypothesis consists in the introduction of an eccentric whose center rotates around the Earth. Such a movement has the effect of making the apogee mobile. Thus, the models *presented* by *Theon no* longer allow us to speak of a characteristic apogee direction

l' for different stars. As Ptolemy , takes the trouble to point out.

54 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p. 2s i, r2fi; Theon of *Smyrna, £x sition,* translation by J. Delatoe, p. J 70 {J 7.3, 1.23].

55 Theon of Smyrna, *Exposition,* translated by J. Dupuis, p. 283, 1.25-30; Theon of Smyrna, Upuxifioa, translated by J. Dclaitrc, p. 171 { i 7•i, 1. 15-20],

J. Dclaitrc, p.1 71 { i 7•i, 1. 15-20],

56 It is, of course, the opposite that is true when the stars move from the perigee to the apogee of their orbit.

57 Theon of Smyrna, Mpo,rïriiio, translation by J. Dupuis, p.281, 1.27; Theon of Smyrna, £rposition, translation by J. Delattre, p.170 [173, 1.23-174, 1.2].

58 Theon of Smyrna. *Exposition,* translation by *1.* Dupuis, p.285,f.22-p.287,1.S; Theon of Smyrna, *Exposition,* translation by J. Oelattre, pp.172-173 [176, 1.22 - 177, 1.7).

59 The difference between the speeds of the astrc on the epicycle and the latter on the detërent, in the context of epicyclic hyperegy, has the same consequence. This is simply even more evident in the context of the eccentric hypothesis.

60 This reflects the fact that the planets reach their minimum—and maximum—luminosity regardless of their position on the ecliptic. As we will see, Ptolemy's models lead us to speak of fixed apogees.

demonstrate,[6] ' that the eccentricity is animated by such a rotational movement around the earth is nothing less than the condition under which the eccentric hypothesis can *remain* equivalent to the epicyclic hypothesis in the case where, precisely, @ z . In other words, the difference in velocities in the epicyclic hypothesis corresponds to the rotation of the eccentric in the other model. Theon does not demonstrate this, does not mention it, and does not even seem to see it: he does not point out the rotation of the eccentric at the same time as he indicates the other essential characteristics of the planetary models, including the difference in velocities and ey. At most, he insists that the above conditions make it possible to account for the phenomena: he indicates that there is equivalence, but not that there is correspondence between these aspects of the two rtiodels.

However, Theon draws other parallels between the two hypotheses. He indicates that the circle described by the center of the eccentric and the epicycle are of the same size (6)° —implied, for a deferent and an eccentric of the same radii. Although he does not say so explicitly, it seems likely that he considers the speed of the eccentric center to be equal to in the epicyclic hypothesis, and the speed of the epicycle to be equal to in the deferential hypothesis. it seems likely that he considers the speed of the center of the eccentric to be equal to in the epicyclic hypothesis, and the speed of the planet on the eccentric to be equal to . This is all the more likely since he specifies that the speed of the center of the eccentric is different from that of the planet on the eccentric(6) '— as he specified / .

chapitre

It is therefore clear that Theon's models are slightly more complex versions of the epicyclic and eccentric hypotheses discussed above. *As we* shall see, the models developed by Ptolemy are much more complex and can no longer be described in terms of a choice between *eccentric* and epicyclic hypotheses. We will nevertheless begin by presenting the main features of the model.

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for **most** planets, but these are the apogees of their *deferents,* not the maximum distances **of the** *planets themselves.*

h l Ptolérnee, ***l'3lmageste,* IV, 5,** translation by G.4. Tnoirrer, p.181 & pp. J **d8-190.**

62 Theon of Smyrna, Ersizzp, translated by J. Dupuis, **p.*85, 1.37;** Theon of Smyrna. *Wpositlon,* translated by J. **Detattrc, p.172** [177, 1.]

63 Theodore of Smyrna, 6xpos/r/ a, translation by J. **Since, p,2S7,** 1.2-3: Theodore of Smyrna, *Exposition,* translation by J. -anc 73(177 of the sun, which we have not yet discussed, although this model corresponds to the simple hypotheses simple hypotheses.

B - movement of pfaziètes of apPtolemy

1 - le soleil

Although the model of the sun has already been discussed at length in Chapter 2, a few reminders are in order. *As* we have seen, the movement of the sun along the ecliptic can be rendered by both the eccentric hypothesis and the epicyclic hypothesis. However, Ptolemy says he prefers the former for its "simplicity," as it involves only one movement instead of two: it can be described by a single circle. He emphasizes throughout Book III of *the Almagest*, however, *that* either *can* be used interchangeably. And, since the movement of the spJeil does not require any refinements— i .e., it has only one anomaly ([5])—it *is* only necessary to determine the parameters relating to it: the average speed, the direction of the apogee, and the eccentricity. Ptolemy explicitly states that these parameters had already been determined by Hipparchus. He then establishes a table of corrections to be applied due to the anomaly.

64 Ptolérnée, *f'Almagesle,* III, 4, translation by G.J. Toomer, p.153.

65 Ptolemy found it easy to demonstrate this: Hipparchus had hypothesized that the tropical year might not be constant, and Ptolemy devoted the first chapter of Book III of *the Almagest to* demonstrating that the errors of his year were due *to a misuse of the observations, and that the tropical year is in fact quite constant, at least on the time scale to which the observations available to him gave him access. (Plolérnée, f'Almagesle, III, 4, translation by G.J. Toomer, p.153.*were due to misuse of his observations, and that the tropical year is in fact constant, at least on the time scale available to him from his observations. (Ptolemy, *Almagest,* fff, 1, translation by G.J. Toomer, pp. I 30-141).

66 For his calculations related to the demiemination of average speed, he cites, as confirmation, certain *observations on* the stars on which Hipparchus, Ptolemy, *and Almagest* based their work, *ie,* III, 4, translation by G.J. T‹›i›irvr. p.199. Regarding the deviation of the apo@éc and the value of the eccentricity: Ptcléni c, *I'AImagerie,* IfĪ, 4, LraJucficin J^

G.J. Toomer, p.153.

Model of the sun

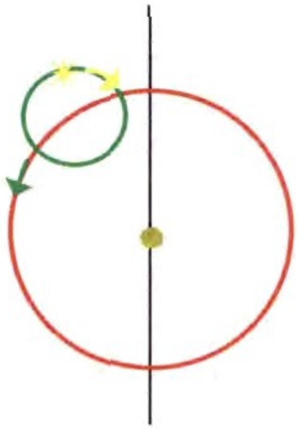

Fig.3.1

In the case of the sun, obtaining the average speed amounts to calculating the length of the tropical year. To do this, Rhoemetea bases his calculations on observations separated by several centuries: two observations of the autumn equinoxes and two of the spring equinoxes, made 285 years apart.[6] He confirmed these with two observations of the summer solstice, 571 years apart—but he was not entirely confident in one of them. [6] He thus concluded that the tropical year is shorter than 365/4 days, by about 1/300.

He therefore finds a value for the tropical year of 365.1448 days." jour.

As we saw in Chapter 2, Theon claims que that the zenith of the orb is located in solaire ♊ 5½ — and donc its base is in ♐ 5½. Il donne the impression of basing pour this on observations. The sun seems to reach its minimum à savoir le speed around ♊ 5½.[70] Ro l émée, using almost the same figure as the one referred to by Theon, demonstrates the value of this apogee, as well as the eccentricity of the orb. Furthermore, he starts from the same data as

67 He holds the oldest ones from Hipparchus, **based on the observations he** trusts. They date back to 46 and 143 **AD**. **The most reliable were** made **by** Rhoemete himself in **139 and 140 (Ptolemy.** t t f.4/mnges/e. III. 1, translation by G Toomer, pp

68 The one carried out by **'the school of Meton and** Euktemon' in - 431 The two of liii. in 140. (Rolémiie, *l'. llniageste.* III, 1, **translation by** G **Topmer, p.138)** ième est bien sur à nouv

9 Pto cmdc *naB esti* iaduct on of G Timmer, p.140

70 "Thus, the sun in v **will be at its greatest distance from** the earth a n d |] will appear to us to have the minimum **of** magnitude **and speed;** this phenomenon appears to occur around 5½ degrees of Gemini. **Thëon of** Dupui 255

71 **Ptolemy,** *l' 4lmngeste* III, 4, translation ‹k; G *l.* Toomer, p.154

translation ike J Dupuis, p.252: see above chapter 2, Fig.5 , Fig.3.9; Théon de Smyrne, *Exposition*, Theon. These were the ones used by Hipparchus, which he claims to have verified himself: the time between the spring equinox and the summer solstice is 94/ days, and that between the summer solstice and the autumn equinox is 92' .

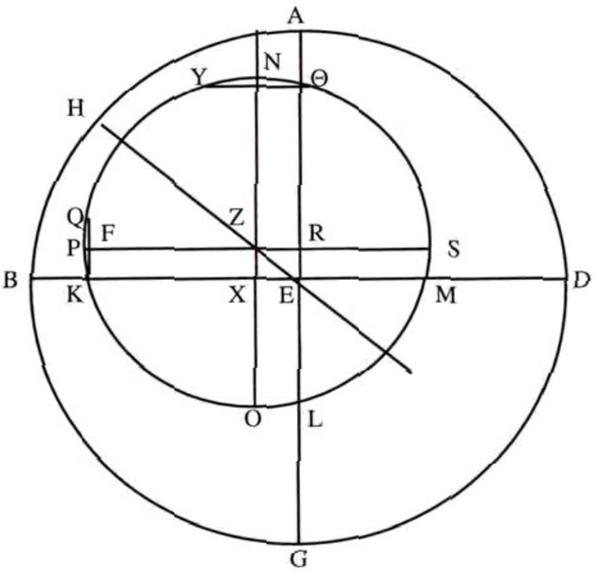

Fig. 3.2

In this figure, the Earth is at E, the center of the eccentric is at Z, so the apogee is at H. B represents the summer solstice. Like Hipparchus before him, Ptolemy thus obtained, by geometric reasoning, the same position for the apogee as that given by Theon: 24 2º ahead (i.e., to the west) of the summer solstice, 2which, by definition, is at the beginning of Cancer. This is II 30; we therefore obtain for the apogee H 30 - 24' 2= fi 5'+2 In the same calculation, Ptolemy also obtains the eccentricity, i.e. the value of EZ: 2;29'/t'.(73)

72 Ptolemy, *Almagest,* III, 4, translation by G.J. Toomer, p.155.
73 Ptolemy, *Almagest, III, 4. Translation* by G.f. Toomcr, p. i55, H2.36. *Taking* as a unit, eoi always, the sixtieth of the diameter of the eccentric, which, following Toomer, we will denote". In the *Book of Hypolhè.ses*

After demonstrating a number of properties—which we outlined in Chapter 2—Rolémée then sets out to show how, from two of the following angles, we can obtain the third: position la moyenne, (E0Z in the eccentric hypothesis and F,-1Z in the epicyclic hypothesis), the observable position, (respectively EDZ and the difference between E W and FDZ, AZO) and the anomaly equation, (respectively DZO and bz).[74]

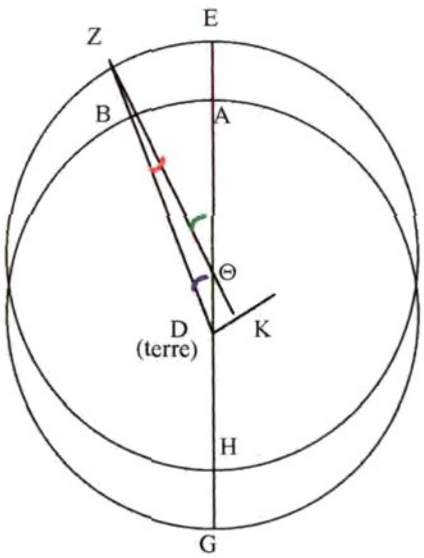

Fig. 3.3.a

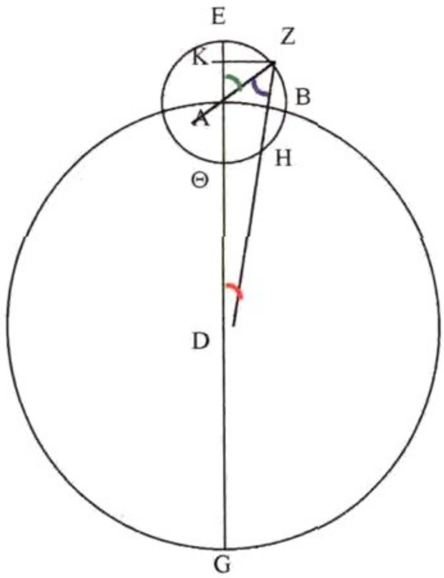

Fig. 3.3.b

Ro1£rr›êe rounds this value to 2: 30ᵉ (Rolémée, le *Livre des Hypothèses,* translation from Arabic by R Morelon, int//D£O, 21 (1993), p.26, 1.6)

74 Rolémèe. / *Almagest,* III, 5, translation by G.f. Toomer, pp.157-166. The figures given below are similar to those described by Ptolemy, but he uses slightly different ones depending on which of the kings' calculations he is performing (Fig. 3.3.a here corresponds to Fig. 3.12 in Toomer's translation, and Fig. 3.3b to Fig. 3

In addition, he performs the demonstrations both according to the eccentric hypothesis and according to the epicyclic hypothesis. For the reader, the figures on which he bases his reasoning seem to represent an arbitrary position of the epicycle, as they do not indicate the direction of the apogee. In fact, this is not the case since, in modern terms, the angular velocity of the epicycle on the deferent and that of the planet on the epicycle are equal. The center of the epicycle is therefore necessarily at the same angular distance from Cf 5W as the apogee of the epicycle.

l l' l' i le (E).

Ptolemy thus obtains a table of anomaly equations. *This* allows him to obtain the position of the sun by tabulating the equation that must be added or subtracted from the average positions of the donkey.[7] All that remains is to calculate what he can then use as initial conditions: the average position of the sun at a given moment, its "epoch," which he chooses as the first day of the year in which Nabonassar's reign began. [76] He obtains H 0;45 for this position.

2 - **The planets and the superior planets (Mars, Jupiter, Saturn)**

We now turn to the models developed by Ptolemy himself, which form the core of his work. It should first be noted that the reasons why Ptolemy opted for certain aspects of these models are not always clear. He himself was aware of this and attempted to justify his choices. D explains that it is difficult for him to explain these reasons, clearly because these choices were the result of trial and error:

> ...if we are at any point compelled by the nature of our subject [...], to make some basic assumptions which we arrived at not from some readily apparent principle, but from a long period of trial and application [...], we may be allowed to accede (to this compulsion), since we know that this kind of inexact procedure will not affect the *end* desired, provided that it is not going to result in any noticeable error...[7]

75 Roléin£e, *l'Almagesle,* III, 6, translation by G.J, Toomer, p.167.
76 That is, February 26, 746, according to Toomer. ‹*Ci.1.* Teomer, *Ptoîemy's Almagesi* p.9). In ?e Zirre *des Zfypatheses, he* uses a different date: 'the first year after Alexander's death, at the beginning of the Coptic month of Toth, 1 noon in Alexandria'. (Ptolémfe, ïe £ivr« *des Hypothêsec* translation by R. Morek›n, *in MIDEO,* 21 f1993), p.50 (for example)).
77 Ptolemy, f'd/magesfe, *Œ* 2, *translation* by G.J. Toomer. p.422. Ptolemy is more explicit about the motivations behind the various aspects of the model of the moon than he is for most other celestial bodies. This seems to be due to the fact that for this luminary, he relies more on the work of his predecessors, particularly that of Hipparchus. Having no way of indicating their respective contributions, he first presents Hipparchus' lunar model with *one anomaly,* then presents his own proposals as a complication of the latter, justified by the fact that it cannot account for certain phenomena.

As we have said, we will begin by presenting the models of Venus and the superior planets, for reasons of simplicity. These are very similar, to the point that practically only a few parameters differ. Like all models except that of the Sun, they differ fundamentally from those known to Theon in that they combine the characteristics of the eccentric hypothesis with those of the epicycle. As a first approximation, they can be described as follows: the other *moves* on an epicycle, which moves *on* an *eccentric* deferent. However, for Venus, Mars, Jupiter, and Saturn, the movement of the center of the epicycle is not uniform when viewed from the center of this deferent, but only when viewed from another point, also located on the line of the apsides (E). This point is on the same side as the center of the eccentric, twice as far from the Earth as the latter. Ptolemy associates it with a circle (of the same radius as the deferent) of which it (R) is the center. This additional circle, called the "equant," therefore has no other purpose than to represent the uniformity of the epicycle's motion, which it does not carry. It should also be noted that, unlike the models of the moon and Mercury, the line of the apsides is fixed relative to the stars: neither the center of the deferent nor that of the equant are animated by a rotational movement around the Earth—except for the same movements as the sphere of the fixed stars, diurnal and precessional. On the other hand, since the hypotheses of the eccentric and the epicycle are no longer alternatives but come into play simultaneously, the size of the epicycle is of course independent of the eccentricity. It now constitutes an additional parameter, specific to each planet, which Ptolemy must calculate.

In addition to these differences from the models presented by Theon, Ptolemy's models also have some similarities. On the one hand, both the movement of the epicycle on the deferent and the movement of the planet on the epicycle are backward, from west to east. *This* is true for all five planets—including Mercury—but not for the moon, as in Theon's model.

On the other hand, the angular velocities of the planet on the epicycle and of the epicycle on the deferent are not equal: r — certainly the most significant difference from the model of the sun. This has an important consequence for Ptolemy, who wants to be able to calculate *the* positions of others: a given position of the epicycle on the deferent does not correspond to a given value of *the* anomalistic position—i.e., a given location of the planet on the epicycle. This implies that, in order to determine longitude positions, we cannot associate an anomaly correction with each mean position, as Ptolemy did for the sun in his table of anomaly equations. He must therefore treat the two movements separately and determine anomaly values that he will count from the anomaly position at the time.

Ptolemy had already inherited Hipparchus' estimates for the average speeds "in longitude," which in his models correspond to the movement *of* the center *of* the epicycle, and even for the average speeds "in anomaly" — which in a simple epicyclic model would correspond to the movement of the planets on their epicycles, for the superior planets.[9] These estimates, which Ptolemy seems to attribute to Hipparchus himself, had already been obtained by Babylonian astronomers, at least to a certain degree of accuracy. Of course, the Babylonians did not see these as movements of planets on epicycles and other elements characteristic of Hellenistic astronomy. The anomalistic motion was associated with synodic phenomena. A return to anomaly was a return to a given position relative to the sun. *Returns "in* longitude" were considered in relation to the stars. Thus, it was known that, for Venus, for example, five returns in anomaly took place in approximately eight tropical years. Ptolemy's contribution to this question was therefore to improve these data. However, the way in which he did so is problematic.

Before moving on to the following models, we will examine in more detail the parameters that characterize the movements of each of the planets, that is, for the models that

78 Ptolemy, ***/'Almagesfv,*** **IX, 3,** translation by Ci.J. Toomer, p.423.
79 As we shall see, **the opposite is true** for **the** inferior **planets**.
80 G.J. Toomer, *Ploleniy 's Almagesl,* p.423, note 19.
81 phenomena related to the position of the star considered relative to that of sr›)eL
82 Pt‹ilémée, *l'Almagesie,* IX, 2 & 3, translation by G.J. Toomer. p.420 **& p.424.**
83 Indeed, **Tocner** redid the calculations as indicated by Piolémée and **obtained results** for the five planets **that differ from those** listed by the astronomer (these results are listed in advance in Book IX, Chapter 3, before Ptolemy gives the **various** demonstrations). **Consequently,** Toomer doubts that he **arrived** at his results in the manner he **claims,** to the point that he devotes an appendix to the question (G.4. Toomer, ***Ptolery's Almagest,*** appendix C**, pp. 669-672).**

occupies: the position of the apogee, the eccentricity, the size of the epicycle (for a deferent with a radius of 60^e), the velocities, the mean and anomalous positions at the epoch. To this end, we must distinguish the case of Venus from that of the superior planets.

a) Venus

There is indeed an important difference between Venus and the other three planets: the latter can reach any elongation, while Venus, like Mercury, remains relatively close to the sun and appears to oscillate from one side of the luminary to the other. Ptolemy accounts for this characteristic simply by stating that for Venus, the direction of the center of the epicycle as seen from the center of the equant is always that of the mean Sun. It is on the basis of this assumption that he calculates the size of the epicycle of Venus: this can be determined from the maximum elongation relative to the mean Sun."

Ptolemy first determines the position of the apogee of the deferent, which is also that of the equant, since the centers of the two circles lie on the same straight line and on the same side of the Earth. He thus obtains Q25°—and therefore, of course, J25° for the perigee.

Ptolemy then determines the eccentricity, equal to 1;'Z‹' '[6] (he will give a slightly lower value lower value in the *Book of Hypotheses* — perhaps rounded: 1 [7]) and the radius of the epicycle equal to 43'/t "The eccentricity is therefore about half that of the sun (2;29' 2)." On the other hand.

84 The calculation is complicated by the fact that, as the epicycle moves along an eccentric deferent, it does not remain at a fixed distance *cousins ïe öe* the earth, so its apparent size varies.
85 Ptolemy, *Almagest,* X, 1, translation by G.J. Toomer, pp. 469-470.
86 Ptolemy, *Almagest,* X, 3, translation by G.J. Toomer, pp. 473-474, p. 474, H306, 1.2.
87 Ptolemy, *Almagest,* X, 2, translation by G.J. Toomcr, pp.42Ofi72; Pt‹ilfmëe, fe £ivrr Jcs *Hypathêses,* translated from Arabic by R. MoreJori, in *MIDEO,* 21 (1993), p.38, 1.10- 12.
58 Ptolemy gives the same value in his book *Hypotheses* (Ptolemy, *Book 1 of Hypotheses, translated by R.* Vorelon, in *3JJD£O,* 2 l f 1993), p.40, 1.1 -2).
89 Result on which he makes no comment,

the epicycle is gigantic: its radius is more than two-thirds that of the deferent! Respecting these proportions, the model can be represented as follows:

Model of Venus

Fig. 3 4

Ptolemy also calculates the position of the center of the equant. *As* we have said, he finds it to be twice the distance from the center of the deferent; in fact, he considers this not as an assumption but as a result obtained by calculation from observations.

In order to calculate the positions of the planets at any given moment, Ptolemy must also determine what corrections to make to his average positions due to anomalies. For Venus, since the center of the epicycle is always located in the direction of the mean sun, the longitudinal velocity is therefore equal to the mean velocity of the sun. Similarly, the average position of Venus at that time is of course the average position of the sun: H 0.45.

90 Rolcméc. f: l/mripr.src. X. I, translation by G.J. Toomcr. pp.472—f74.

As we have seen, the way in which he obtained more accurate values for the anomaly velocity is controversial. In any case, he obtained for Venus 1800" parcounis in anomaly in 29ł *9; 40* days, or 0; 3ó, 59, 2J, 53, 11, 28º per day. To obtain the position in anomaly at the epoch, Ptolemy simply considers the position in anomaly at a given observation and subtracts the difference in anomaly corresponding to the time elapsed since the epoch at the time of the observation. This gives a position of 7 l; 7e (counted, as always, from the apogee of the epicycle)."

b) Mars, Jupiter, Saturn

In addition to the parameters specific to each planet, the model that Rolémée uses for the superior planets differs from that of Venus only in its position relative to the mean sun. While for Venus, the direction of the center of the epicycle was also that of the mean sun, for the superior planets, the angle between the diameter of the deferent and the mean sun as seen from Earth is equal to the sum of the distance traveled [tar 1 epicycle (as seen from the center of the equant^) and the distance traveled by the planet around the epicycle (3):

$A\Theta S = AEZ + Z($

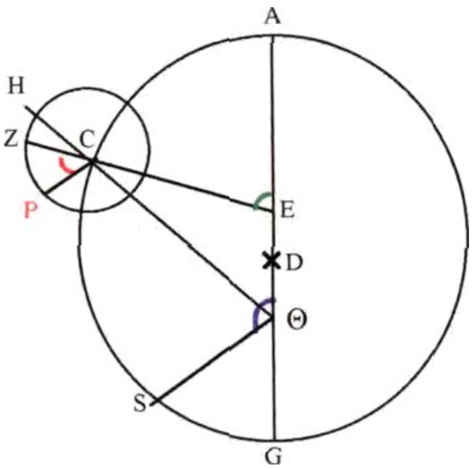

Fig. 3.S

91 Ptolemy. / A/mo;gesfe. X, 5. Translation by G.J. Toomer. p.479,
92 since it is clear from this point that the movement of the epicycle is uniform.
93 seen from the center of the epicycle, of course, ct GomØć ù țørtir from the ajxigće of the latter relative to the center of f'čquant. Roićtnéc. */',lfmageste, X•*. 6. tmductïon dc G./. Tœmęr. p.48ż, H3 f8. î. î ł-HI *19. ì.3.*

This means that the direction of the sun, as seen from Earth, is always parallel to the line joining the center of the epicycle to the planet."

We can therefore represent the model proposed by Ptolemy for the superior planets in the following way:

Model of the superior planets

between the úquiint

di icct ondu
soleil moyen

Fig.3.ó

Thus, for the superior planets, it is the anomalous motion that is linked to the motion of the mean sun, and no longer the mean motion 'in longitude'. i.e.

94 R0lém6e. / ?i /znoprsfe. X, ti. **translation by G.J Toomcr.** p 4S3. H319. 1. 11-H320. 1.3

En effet:

$A\Theta S = AEZ + ZCP$

CEÔ * AO-H+ ÔCE= l8tl (triangular CEO1

$AEZ + CE\Theta = 180$

$AEZ = A\Theta H + \Theta CE$

$AEZ = A\Theta H + HCZ$

$A\Theta S = A\Theta H + HCZ + ZCP$

$A\Theta S - A\Theta H = HCZ + ZCP$

HÕ-S= HCP therefore OS *II C* P

Rolimée. :/ l *liiinge.!w. X.* 6. Translation by G.J. Toomer. p.4BJ. H32f1. 1.4-12.

the epicycle on the deferent as is the case for Venus (and Mercury, as we shall see)

In order to determine the eccentricity and direction of the apogee—which he does in the same way for the three superior planets—Ptolemy uses these aspects of the " Indeed, these imply that when one of these stars is in opposition or in conjunction with the mean sun. It is in the same direction as the center of its epicycle (as seen from Earth). modele.

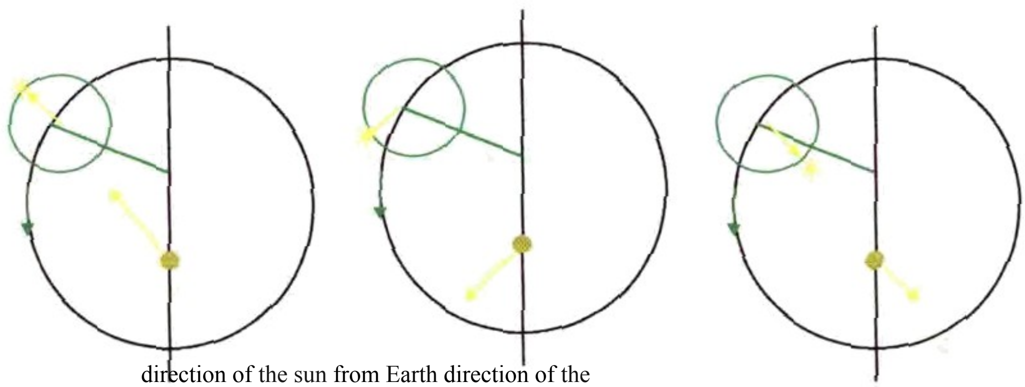

Fig. 3.7

Since conjunctions are difficult to observe, even with the average sun, Ptolemy uses oppositions.^' Unlike what he did in the case of Venus, where he calculated the excentricity directly—i.e., the distance between the Earth and the center of the *deferent—for* the superior planets, Ptolemy actually calculates the distance between the Earth and the center of the equant. He then obtains the excentricity from the latter, albeit in a rather o‹/ /i‹›c The table below summarizes his results.

'75 Rolciisée. / lfnioyc. re. translation by G Toonicr. X. 7. pp 9f›

Ptolènicc. *l'. Ihnapesir.* translation *öc* G Toomcr. X. 7. p

.J. .484-499; XI, 1, pp.507-519; XI, 5, pp 525-538.

.J. .484; XI, 1, p.507; XI, 5, pp.525-526.

	distance between Earth and the center of the equant	position of the apogee
Mars:	12	G 25; 30º “
Jupiter:	5; 30e"	♍ 11° [100]
Saturn:	6;50	♏ 23° [102]

Directions of the apogea of the deferents of the planets
el
Direction of the apogee of the sun's orb

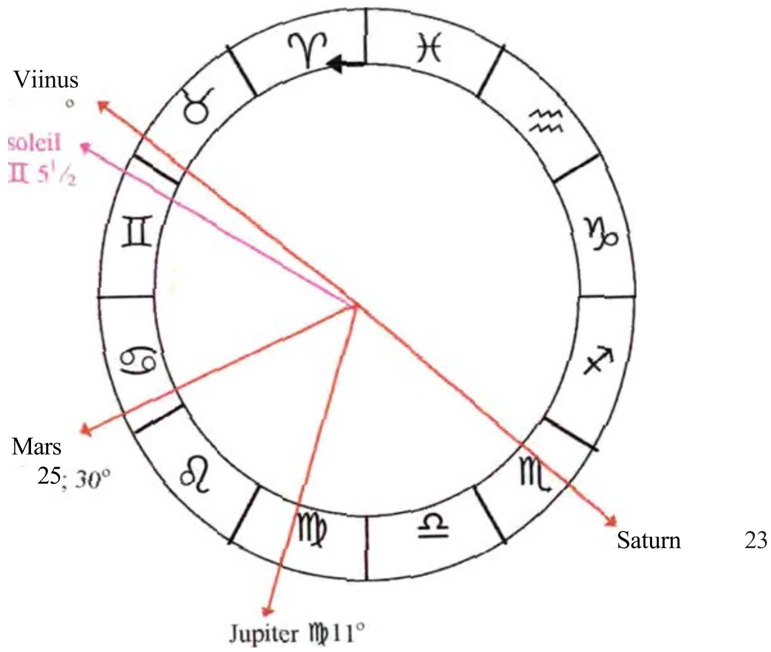

Fig. 3.8

97 Ptolémér,. / ï/mn,ge.Ile. X. 7, translation by G.J. Toomer, p.495: Rolémée, le *Livre* de.s *Hvpolhèse.s.* translation from Arabic by R. Morelon, in *PHDTO,* 21 (1993), p.42, 1. I*-17 (Rolémée indicates that the distance between the center and the eccentric is equal to 6).

98 Rolémét, *l AImagerie,* X. 7, translation by G.J. Toomer, p.498.

99 Roléniée, *l'Almapeste,* Xl, 1, translation by G.J. Toomer. p.515. Rolémée, *le Liare des Hvpothèses.* Translation from Arabic by R. Morelon. In 5MDE‹ . 21 (1993), p.46. 1*.23-25* {Rolémée indicates the distance between the earth and the exoenkliric as being equal to 2. 4S').

100 Ptolemy, *Almagest,* XI, 1, translation by G.J. Toomer, p.519.

101 Ptolemy, *Almagest,* XI. 5, translation by G.J. Tooiner, p.533. Rolcmée. le LÎrre *des Hvpolhèse.s.* translation from the Arabic by R. Mormon, in *!IIDL“O. 2* I (1993). p.52, L6-8 (Ptolemy indicates the *distance* krre-centre of the eccentric as equal to i 3 : 25e).

102 Ptolemy, *the Aimagesia.* XI, 5. Translation by G.J. Toomer. *p.*537.

For the sizes of the epicycles of the three planets, Ptolemy calculates the following values, which he will repeat exactly in *the Book of HjjnothèSex:*

Mars: 39: 30‘ [103]
Jupifer Saturn 11; 30° [104]
6; 30° [105]

If we represent the sizes of the epicycle of all the planets and luminaries on the same figure, in proportion to the size of their deferent, the variation in this ratio from one celestial body to another is particularly striking. [106]

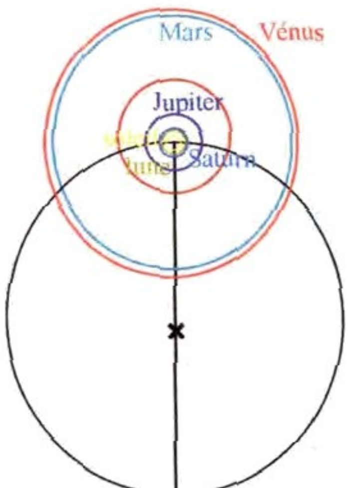

Fig. 3.9

103 Ptolemy, *the Almogect, tmdun ca* by G. J. Toomer. Y 8, p. 501, H3â 1. 1 ~~*Hypaihèses, translation from* Arabic~~ by K Mormon, in *MIDEO,* 21 (1993), p. 44 .17; Ptolémée, *le Livre des*

104 Rotéinée, *l'4lmageste, translation* by £i. *J* Toomer. Xt, 3, p.522, H *Hypotheses* translated from Arabic by R. Mormon. In *MIDEO,* 21 (1993), p. 48. 1.22-124.

105 Roléinée, *l'Almageste,* translation by G. î: Tooiner, XI, 6, p.54O, H419, 1.5; Rolémée, Je *Livre des Hypothèses* translation from Arabic by R Morelon. in *MIDEO,* 21 (1993), p.54, 1.2-3.

1% Mercury's epicycle has a radius of 22: 30 translation by G. Toomet. 578, H488, 1.7). and that of the moon is 5; 14 (e, translation by G.J. Toomet. IV, 6, p.202, H322. 1 4) (Ptolémée, *l'Almageste*, tra

For information purposes, the average speeds of the planets are as follows:

	The average speeds in longitude are:	and in ano mali
		2Y t.40, t 20
Jupiter	0: 4. 59. 14. 26, 46. 31°	0: 54, 9.2, 46, 26, 0°
Saturn	0: 2'', i3. i. 28, 51, 33°	0; 57, 4 43 9, 43, 58°
:	59 . 1 12 (identical to the mean sun)	0; , 7, , 41, , 4 °
Vénus:	0; , 8 7, 13, , 31°	0; 36, 59, 25, 53, 11, 28°
Mercure:	0; 32, 56, 27, 26, 23. 41°	3; 32 29 44 590 45 39° 30

P *nc* ttc five planets: Ptolemy, *l'Almageste,* translation by G. *I.* Toomer. IX, 3, p.424; for the Ptolemaic moon, *the Almagest,* translation by G. J. Toomet, IV, 3, p.179; for the average speed of the sun in æ+‹n « •*Bes* trafhict on & G Toomer. III, 1, p 38

3 - Mercury and the moon

More complex are the models of the moon and Mercury. The essential difference between these and those of the outer planets we have just examined is that this time the direction of the apogee is no longer fixed: the line of the apsides rotates because the center of the deferent is rotating. This resembles the *eccentric* model proposed by Theon, except that, of course, in the latter there was no epicycle—and therefore, of course, no equant. This peculiarity has the effect that the distance of the epicycle from the earth varies not only according to the position of the deferent but also according to the position of the epicycle. there was no epicycle—and therefore, of course, no equant. This peculiarity has the effect that the distance of the epicycle from the earth varies not only according to the position of the epicycle on the deferent, but *also* according to the position of the center of the deferent on its circle. This is Ptolemy's motivation for introducing this system."

Furthermore, if in both cases the movement of the epicycle on the deferent is once again backward—i.e., from west to east—the center of the deferent rotates in the opposite direction, forward—from east to west. The movement of the other on the epicycle is backward in the case of Mercury and forward for the moon, as in Theon.

The Nlercure model includes an equant—a circle from the center of which the epicycle moves uniformly—but this equant is not positioned like those of the superior planets and Venus. For these, the center of the equant was twice as far from the Earth as the center of the deferent. Since the latter point is no longer fixed in the Mercury system, we consider the position of the center of the circle on which it moves (Z). The center of the equant is then located in the same direction at mid-distance from this point (Z) according to *1'Almnge.ne,' "*and at 6/1 1'" according to *my Book Je.s.*

Iîî7 Rolcméc. *I \4 litiapeste,* translated by G J Tœmer, V. 3. pp.22tj-22 I: IX. 5. p.443 lfJfî
Roldmée. *I Ùltnugexle.* translated by G. J, *Tœiner.* V, 3. pp.23f›-37ï
IUS Rolêméc. *I illrtiage.ste.* translation by G. J. Toomcr. IX. 3, p.4-t9. H2'î1. I th
Rolùméc. /'d/1nnye.sfc. translation by G. J, Tuner. IX. 6. p,•I-Î4.
I I ï Rolémée. *I'.4lt»nge.sfe.* translation by *f3.* /. Toomer. IV. *5. pp. t fitfi•I8û.*
112 Rolëmés. / *:1Iniage.ne.* Introduction by G J Tœmer. IX, fi. pp.444-445.

NJy›/N•.›e.r," a difference that indicates that Ptolemy continued to improve his models after *Alnuige.vfe* — and did so in the direction of greater complexity where necessary.

Modèle de Mercure

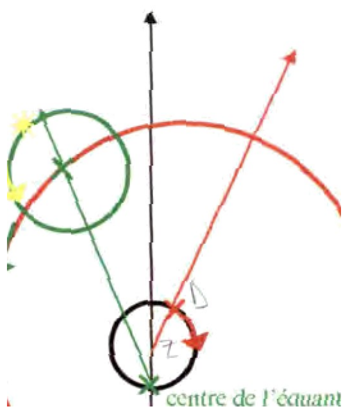

Fig 3 I O

We note that, since the center of the equant is fixed, its apogee is also fixed, unlike that of *the deíècent.* There is therefore something in this model that gives it a definite orientation relative to the ecliptic, which is not the case with Theon. This is all the more true since it is also in this direction that the point around which the center of the deferent rotates is located. This last point means that the center of the deferent (D) is at its maximum and minimum distances from the Earth when it is in this direction. When it (D) is at its maximum distance, the epicycle is then located at the apogee of the deferent, and is

113 Ptolcinóe. le /.irm ‹f.c /(tpr›zúJ.sr.s. translation **of the;iratc by R.** Morclon. in*:kIIl1k.í1*. 21 (1993). p 34. 1.1 H. The distance tcrre-Z is in fact **equal** to 3" and the distance Z-E to 2: 30", the circle on which the **center of** the deflector moves **for a distance** of 2: 3t).

then at the greatest distance from the earth that it can reach. At this moment, the epicycle is also in the direction of the apogee of the deferent. If we consider a complete revolution of the epicycle, we find, however, that we cannot define a perigee of the epicycle in this way: it is never at the perigee of ***the deferent*** when the latter is closest to the Earth. In fact, for each revolution of the epicycle, there are two configurations in which the latter is as close to the Earth as it ever gets: for the Mercury model, there are two perigees, on either side of the perigee of the equant. ([4])

On the other hand, Mercury has this in common with Venus in that its elongation relative to the Sun is limited (hence its classification as an inferior planet). As with Venus, Ptolemy translates this by assigning the direction of the mean Sun (as seen from the center of the equant) to the center of the epicycle. Therefore, as with Venus, and unlike the superior planets, it is its mean position 'in longitude' that depends on the Sun and not its anomaly—i.e., the position of the planet on the epicycle.

In the case of the Iune, the point around which the *center* of the deferent rotates *is* none other than the Earth. We therefore have a constant value for the eccentricity, equal to 10; 19 .' '" The model of the ltine is also simpler in that it does not include an equant. The movement of the center of the epicycle is certainly not uniform with respect to the center of the deferent along which it moves, but it is uniform when viewed from the Earth. In fact, we can say that in the model of the moon, the Earth plays the same role as the center of the equant in the models *we* have seen above. This nevertheless constitutes a violation of the principles laid down by Plato, if we understand these as requiring circular and uniform motion as seen from the same point. In *the Almageslv*, where the model of the moon is presented first, this is the first case of violation of these principles committed by Ptolemy. He makes no comment on

114 Ptolemy, *Almagest,* IX, 5, translation by G.J. Toomcr. p.449: H253.
IS **Ptolemy, /** ***'Almugesie,*** **V, 4,** translation by Ci.J. Tuomer, pp.225-226; **Piolémce** gives a higher value, equal to 12; CO in the *Book of Hypotheses (*Ptolémfic, fr l‹vre de.s H*;*poïhêses,* translation from Arabic by R. Morelon, in *MIDEO,* 21 (l9S3)), p.28, t.14.
1 t6 The other will be the introduction *of* the quantifier,

this decision *Furthermore,* the new moon on the epicycle is not uniform in relation to the true apogee, i.e. the point of the epicycle furthest from the earth. The movement of the celestial body is only uniform in relation to the *"mean* apogee." This is defined as the point on the epicycle furthest from the point on the circle of rotation of the deferent that is diametrically opposite the center of the deferent.

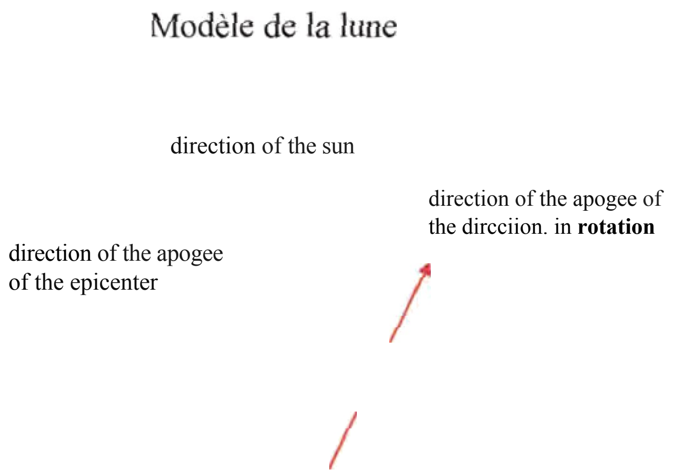

Fig.

In the model of the moon, it is not possible to define a fixed orientation relative to the ecliptic as we could with Mercury, despite the motion of *cotation*.

117 The furthest point of the epicentres intersected by the Earth-centre line for the apogee.
118 Rolciiicc. / !1/z»,vr.si '. ¥'. ü. translation by G.J Tooincr. pp.22s›-227

from the center of its deferent: in fact, the two points that allowed us to draw it in the case of Mercury (the center of the equant and the center of the circle described by the center of the deferent) both correspond to the Earth in the model of the Moon. On the other hand, the orientation of the system is easily defined in relation to the mean Sun, which makes it possible to account for the phases of the Moon. Because the center of the deferent and the epicycle rotate in opposite directions, Ptolemy uses the concept of double elongation: at new *moon, the center* of the epicycle and the apogee are both in the direction of the mean sun; they then move away from each other, each forming an approximately equal angle with the direction of the mean sun.

Ptolemy is more explicit about the motivations behind the different aspects of the model of the moon than he is for most of the other celestial bodies. This seems to be due to the fact that for this luminary, he relies more on the work of his predecessors, particularly that of Hipparchus. He therefore first presents the lunar model with a single anomaly already known to Hipparchus. The second anomaly (i.e., "erection") appears as a complication of the first, justified by the fact that it cannot account for certain phenomena. Indeed, Ptolemy explains that while the first anomaly is sufficient, or almost sufficient, to account for the positions of the moon at syzygies, this is not the case at quadratures: the anomaly observed there is much *greater than* predicted, i.e. if the moon is then located east of its mean position, it is further east than predicted.

— and similarly in the West. Everything therefore happens as if the effect of the epicycle were amplified at the quadratures."[0]This can be explained if the epicycle is then closer to the Earth; Ptolemy therefore hypothesizes an eccentric deferent ([2])— which causes the distance of the epicycle from the Earth to vary — rotating over a period of one month — so that this distance depends on the position of the moon relative to the sun·([2])

119 Ptolemy, *Almagest,* V, 2, translation by C.J. Tooiner, pp. 220-221.
120 Ptolemy, *Almagest,* V, 2, translation by G.J. Tooiner. p.220, H3S5.
121 Ptolemy, *Almagest,* V, 2, translation by G.J. Toomer, p.220, H355, 1. t8--356. 1.3.
122 Ptolemy, ***f'A/ma,qe.six,*** V, 2, translated *by* G.J. *Toomor,* ***y.22s,*** Hfi56, f.9- *IS.*

IV - **on the movement of pplanets in latitude**

The movement of the planets in longitude, which must be accounted for by the models we have just presented, was certainly the main problem facing Hellenic astronomy. Nevertheless, it also had to account for their movement in latitude, a task that was not necessarily any easier. Here again, Ptolemy's contributions are very substantial, although they vary considerably between *the Almagest* and the Book of Hypotheses. The known models of Theon appear extremely rudimentary in comparison.

Let us first examine the circle in relation to which the two astronomers consider the movement in 'latitude' (in the broad sense): when they refer to a movement approximately orthogonal to the one we discussed earlier, are they actively referring to the latitudes themselves (i.e., distances from the ecliptic), or to the declinations (i.e., distances from the equator)? The first time he addresses the question, Theon considers this movement in relation to the equator. He says:

> ...they [the stars] have *a movement in latitude,* from north to south and vice versa, while completing their course in the opposite direction to the movement of the universe. Attentive observers see them carried from *summer tropic to the winter tropic* and vice versa, across the obliquity of the zodiac. 23

He also refers to this same movement on either side of the equator, considering it a movement in "latitude" in the broad sense.

123 Theon of Smyrna, *Mpnsition,* translation by J. Dupuis, p.221, t.22-27. mes itaJ iques.

However, in other places, Theon also considers the movement in latitude in relation to the ecliptic. 24

This is quite different from what Ptolemy does, who, when expounding the theory of latitudes for the five planets, states from the outset that these are considered in relation to the ecliptic.

One of the most surprising elements in Theon's work is that he attributes to the ecliptic a movement in latitude relative to the ecliptic. In Ptolemy, such an idea would be almost a contradiction in terms: "It [the ecliptic] is precisely defined and, so to speak, drawn by the motion of the sun."(27) The way in which Ptolemy deals with the problem of eclipses leaves no doubt that this is indeed his conception. (2) Why then does Thénn attribute such a movement to the sun? This is all the more curious since, when he undertakes to give the period of return in latitude of the luminary, he refers to a movement considered in relation to the equator. In fact, he takes shadows of gnomons of the same length as a reference to define these returns; since the position of the ecliptic is not *constant* for a given place, this method can provide him with returns in latitudes relative to the ecliptic.

It seems that Theon intends to account for movements in latitude solely by means of an inclination of the circles (eccentric, or deferent and epicycle), inclination relative to the equator, which goes without saying, and relative to the ecliptic, since he recognizes the existence of deviations from the latter. It is likely that he attributes the same inclination to the epicycle and the deferent, and that he conceives them as being located in the same plane, since he does not bother to specify that this is not the case.

124 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.221.
125 Ptolemy, *J'A/œngeizc,* XIII.1, *tr»ôociiDn* by G.J. Toomcr, p,597, 1.2.
126 Theon of Smyrna, *ExpOSftfon,* translation by J. Dupuis, p.223. 1.8-9 (somewhat unclear ï; Theon of Smyrna, iqu.milieu, translation by J. Delattre, p.129, [135, 1.14- 1>l
127 Ptolemy, / *Almageste*, 1.8, translation by G.J. Toomer, p.46.
128 see chapter 4, 1ère part.
1 *9 Thfon of Smyrna, *M,yosiiion.* translation by I. Dupuis, p.2f›1, 1- 11 -1 5. 110
Theon of Smyrna, frpurifion, translation by J. Dupuis, p.28 l & p.297.

Theon points out that the five planets do not stray as far from the ecliptic in the north as they do in the south. This is difficult to explain in a model as simple as Theon's—as we shall see—if the eccentric is divided by the ecliptic into two equal parts. It is therefore reasonable to assume that the intersection between the eccentric and the ecliptic passes through the Earth and not through the center of the eccentric

— although Theon says nothing about this. This is all the more likely since, except for the sun, the center of the eccentric is rotating. In the epicyclic hypothesis, the intersection between the deferent and the ellipse would pass through the center of the deferent.

Theon seems to conceive of this inclination as fixed for each astte, although typical of each of them. Furthermore, he does not seem to attribute any rotational movement to the nodes, the points of intersection between the deferent (or eccentric) and the plane of the ecliptic (which he does not define). In fact, he makes no reference to such a movement. It is true that the period of return in declination that he gives for the sun is 365 days 1/8, which is different from the period of return in longitude (365 days 1/4, of course), and that the simplest way to interpret such a difference would be to postulate a rotation of the nodes. However, Theon does not insist so much on this difference as on the fact that it is small. He therefore seems to neglect it. Moreover, in the case of the five planets, it is implicit that each reaches its maximum latitudes on either side of the ecliptic always in the same direction. Theon states that each of these planets "in one sign is very far north, in another very far south" ([32])or "they are more northern in some signs, more southern in others." Ptolemy shares this view, at least for Venus. Mars, Jupiter, and Saturn. However, this is not the case for the moon. Although Theon does not specify anything on this subject, it is difficult to imagine that he was unaware of the behavior of the moon, which can reach its maximum latitudes at any point on the ecliptic: the variations in latitude of this celestial body are so marked that it is

19 I Thdo \ dc Smymc, *Exf›OSiion,* Translation by 1. Dupuis, p.221, I. I 3- I6. "
192 Theon of Smyrna, *Exposition,* translation by J. Dclattre, p. I29.
I ñ1 Thdon de 8inyrnc, *Exposiiinn,* translation by 1. Dupuis, p.221.

It is inconceivable that the phenomenon could have gone unnoticed. Theon's silence on this subject is therefore most certainly a simple oversight.

The model described by Theon to account for movements in latitude therefore seems to be as simple as can be. Olaf Pedersen believes that it was known to Hipparchus, basing this on Ptolemy's statement that '...lit.e Hipparchus, wc *rnd* by calculatioii from the moon's most northerly and southerly apparent positions that its greatest deviation either side of the ecliptic is approximately that amount [8º).' In fact, this passage does not refer to a given model, but given the simplicity of what is involved, we can indeed assume that he calculated *these* maximum deviations in order to determine the angle between the ecliptic and such an inclined circle on which the moon or its epicycle would move.

As we have said, Ptolemy's theory of latitudes is much more elaborate. One of the fundamental differences is that, once again, he proposes different models for different celestial bodies: namely, one for the superior planets, one for Venus, one for Mercury, and another for the moon. All have an inclined deferent. And, remember, Ptolemy describes this inclination as being "in relation to the ecliptic."* He specifies that "the intersection between the deferent and the plane of the ecliptic passes through the Earth," which implies that it does not divide the deferent into two equal parts, as in Theon's model. He calls the northern and southern limits the points on the deferent where the latitude is highest—north and south of the ecliptic, of course.

This time, the simplest model is that of the moon. In fact, it differs from that presented by Theon in only one respect—which is typical of the case of the moon: the nodes are animated by a rotation from east to west, of 0.3° per

t fi4 O. Pedersen, 4 .i«/v *of the Almagest,* p.357; **Pedcrxrn** refers to **page** 20fl of his work, where he **then gives** the reference for the passage from *Almagest* on which he bases his argument.
135 Ptolemy, *Almagest,* V, 7, translation by G.J. Toomer, p.237, H3S8, 1.18-20. 1 36
Ptolemy, ***Almagest,*** XI II.l, translation by G.J. Toomer, p.597, 1.2.
1 57 Ptolemy*, Almagest,* XIII.1, translation by G.J. Toomer, p.597.

day.''' For *the* rest, the model simply consists of a 5" inclination of the Referent relative to the plane of the ecliptic.''' According to Theon, this value of û" differs from that accepted by 'most mathematicians', who estimate it at 6".'''

Apart from the moon, Ptolemy considers the nodes—and therefore the northern and southern limits—to be fixed. In the case of the inferior planets, the axis of the limits has the same direction as the apogee-perigee axis of the deferent. This implies that it is fixed for Venus but not for Mercury: since its line of apsides is rotating, its limits cannot be fixed relative to the stars. This does not correspond to what Theon proposes, for whom the nodes—and therefore the limits—appear to be fixed for all planets.

For the superior planets, the northern limits do not correspond to the apogee of the deferent—although there is little difference in the case of Mars. Their longitudes are additional parameters specific to each planet:

142

Mars:	(in Cancer
	beginning of Libra
Saturn:	beginning of Libra
Jupiter:	

They are therefore fixed, as in Theon's model."

The figure below summarizes these data, as well as the directions of the apogees of the deferents of the five planets and the sun:

t 38 Ptolemy, *Almagest,* V, 2, translation by G.J. Toomer, p.220, H357, 1. J-4.
139 Ptolemy, f *'Alznagesie, ¥,* 12, translation by G.J. Toomer, p.247, H407, J.8-9; Ptolemy, *The Book of the I-lyyolhëses,* translated from Arabic by R. Moreton, in *MIDEO,* 21 (t993), p.28, 1.10- 1l.
140 Theon speaks of a displacement amplitude in latitude of 12', therefore (since he considers that there is synimètric on either side of 1' of the ecliptic in the case of this astro) 6º of maximum deviation from the ecliptic. Theon of Synchron, *Exposition,* translation by J. Delaitre, p.129. [135, 1.14-18]: the corresponding passage in the translation by J. Dupuis is unclear (p.223, 18-12).
141 Ptolc'mée, *l'Almageste,* translation by G. J. Toomcr, XIII, 1, p.599, H52h, 1,6-g.
142 Ptolemy, *Almagest,* translation by G. J. Toomcr, XI II, 1, p.fi98. H52f›, t.5-7.
143 The fact that it is different.

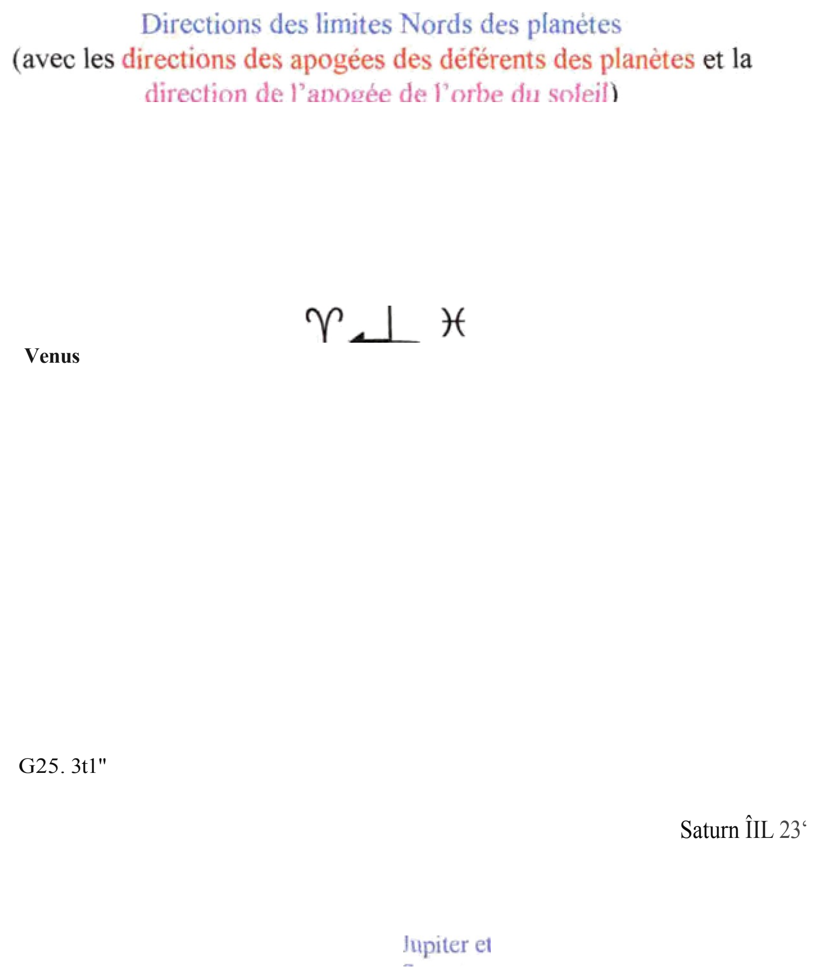

Fig 3 12

Apart from the fact that their northern limits differ from the apogee of their deferent, it is again for the superior planets that the model is simplest. Indeed, it requires that the inclination (i.e., the angle between the deferent and the ecliptic) remain constant. So far, these models are therefore not very different in principle from those of Theon.

— put to part may be their orientation. Rolémée adds another anomaly, however: the epicycle is (generally) not in the same plane as the deferent. From

144 **Roiùmée.** *l '4 hnapesle.* **translation by** G. J. Teomcr. XII1. 2. *p.S•l•1.* Hfi2'7. **1.4-'›.**

plus. The angle it forms with the latter is not constant: the epicycle moves back and forth around an axis passing through its center and parallel to the deferent'[4] " — therefore, in general, not parallel to the ecliptic.'[46] The period of this movement is equal to that of the epicycle on the deferent.'[4] ' When the center of the epicycle is at the nodes, this angle is zero and the epicycle is then in the same plane as the deferent — and the ecliptic. Everywhere else, the angle between the two circles increases the latitude of the star relative to what it would be due to the inclination of the deferent. The angle thus reaches a maximum when the epicycle is at *the limits."* The reason for Ptolemy's choice is that he distinguishes between two anomalies for movements in latitude — as was the case for movements in longitude. Once again, one is related to the position on the ecliptic, in longitude: this is the anomaly reflected by the inclination of the deferent. The other is related to the epicycle. ([4]) [e]The notion that such an anomaly exists stems from the fact that the three celestial bodies have a higher latitude when they are at the perigee of their epicycle. It is obviously to account for this second anomaly that Ptolemy inclines the epicycle, *so that* its perigee has a higher latitude than the center of the epicycle. As we have seen, the models presented by Theon do not distinguish between these two anomalies.

It is also surprising to note that the two astronomers disagree on the maximum observable latitudes, as shown in Table 1 below, which summarizes the parameters of the models of Ptolemy and Theon:

145 Ptolemy, *f'Almagest,* translation by G. J. Toomer, Xil1, 2, p.599, H530, 1.9-12.
146 Ptolemy, *"Atmogest,"* translated by G. *1. Toomer, XIII,* 2, p.599, H.529, J.8- *1 1.*
147 Ptolemy, f'N *Imagerie,* translated by G. J. Toomcr, XIII, 2, p.599, H529.1.13-H530,l.l.
148 Ptolemy, f *'Almagest,* translation by G. J. Toomer, XII I, 2, p.599, H53O,I.3-8.
149 Ptfilémée, f *'Almageste,* translation by G. *1.hoornoi,* Xfff, *1, p.597,* 1.8- *l I.*
150 In fact, Ptolemy only mentions this explicitly for its position in northern latitude, but his model itself implies that the same applies to the south: "the paris of the epicycle nearer the earth are always inclined in the same direction as the eccentric..." Ptolemy, f *'A Image.ste, Xî ïi. I,* translation by G.J. Toomcr, p .59s›. H526,l.10- 12.

	MARS		JUPITER		SATURN	
	Angles	Maximum deviation in lartude cories ondante	Angles	Deviation in latitude corres ondanle	Angles	Deviation Maximum latitude corres ondanłe
In Ptolemy: łncînaison of the dśłéient on the ecliptic"'	1	1°	1;24"' *	1;24	2;26"	2;26
Inclination the epicycle on the deférent following the axis of the epicycle parallel to the deférent	g] 5•^	to 156	2;30° '*7	0.3sO1 '8	4;30° 159	o.s4° '°°
According to Ptolemy in the Hypophases: Inclination of the deférent on the ecliptic	' 5°' "'	‹ : s0°	; ao°1 62	1; 30"	2; 30"	2; 30"
Constant inclination of the epicycle on the deferent following the axis of the epicycle parallel to the deferent constants	4; SO° '		1; 3O"'		2; 30" "	
they Tfiéon: Inclination of the deferent on the ecliptic constant	2;30	2;30	2;30	2;30	1;30	1;30

152 XJIf, 3, pp.6O2-605, H337,l.5-H54*,l.9.
153 Plolémće immediately decides to round to 1.5", Piolérnće, *I'Almagesie,* XI1I, 3, p.605, H542, í. łß-11.
154 Ptolémée immediately decides to round to 2.5", Piolémće, *'Xlmugvste,* XIII, 3, p.605, H542, 1.10-1 1 .
155 Plolémëe,*f'Alrnageste,* translation by G. J. Toomcr, XIII, 3, p.604, H540, 1.17-19.
156 Plolfmće. *I'Almagesle,* translation by G. J. Toomer, XIII, 3, p.604, H540, 1,14.
157 Ptolemy. *Almagest,* translation by G. J. Toomer, XIII, ?, p.6d5, H542, ł,12.
158 Ptolémće, /'Afinagesre, translation by G. J. Toomer, XIII, .3, p.605, H542, 1.6.
159 PfOff,mèe¸ */'Almogesle,* translation by G. J. Toomcr, Xl11, 3, p.605, H542, 1.12..
160 Piolćmće, *I'Almageste,* translation by G. J. Toomct, XIII, 3, p.605, H542, t.5.
161 Piolćmëc, *le Lis re des Hyƒ›olhêses,* in *4f1DEO,, y.4 ż, L9.*
162 Ptolćrrtče, fe Z* ze *de Hypalhèses,* in *MIDEO ,,,* p.46, 1.14- 16.
1G3 Plolémće, *le Litre äes Il)!polhêses,* in *MIDEG,,,* p.50, 1.7-0-24.
164 Ploiétriče, fe *Livre des Hypnlhćses,* in *MîDEO,,,* p.44, f.9- ł û. According to the Arabic version of this work, the inclination of the f'ćpicycfe on the *ãćfćrcni and* the inclination of the dćfčreni on the ecliptic differ in the case of Mars, contrary to what Olaf Pedcrscn states. A *Survey of the Almagest, p* .?92.
165 Ptolemy, *Book of Hypotheses. In MIDEO,* p.48, 1.19-21.
166 Ptolfmëe, *the Book of Hypotheses,* in *M1DEO,,,* p.SP, 1.28 - p.54,1.2.
I ò7 Thfion dc Smyme, *Exposition,* translation by J. Delattrc, p.l 29, [I š5, 1.14-19): the corresponding passage *in* Dupuîs' translation, p.223, 1.8-13, is unclear,

For the inner planets, the back-and-forth movement of the epicycle is in a sense the opposite of that of Mars, Jupiter, and Saturn: when it is located at the limits, it is in the plane of the deferent, and when it is at the nodes, *the angle it* forms with the deferent is at its maximum. [6]The direction of this back-and-forth movement is reversed for the two planets: at the ascending node, the perigee of the epicycle is south of the deferent (and the ecliptic) for Venus, and north for Mercury; conversely, at the descending node, the perigee of the epicycle is north of the deferent for Venus and south for Mercury.[1]

In addition to these differences, the models of the inferior planets are more complex than those of the superior planets because Ptolemy adds two additional anomalies, which are of course entirely absent from Theon's known models, judging by what he reports.

Firstly, the inclination of the deferent on the ecliptic is no longer fixed. The deferent itself is animated by an oscillating motion, with the line of the nodes as its axis. Its inclination relative to the ecliptic is maximum when the epicycle is at its limits, and zero when it is at the nodes: the deferent is then in the plane of the ecliptic. The oscillation is such that the center of the epicycle is always on the same side—north or south—of the ecliptic. Thus, the epicycle is always located north of the ecliptic in the case of Venus, and south for Mercury.[(70)]

Secondly, the epicycle is animated by a second oscillatory movement around an axis passing through its center, its apogee, and its perigee. This movement is such that the inclination relative to the plane of the ecliptic—and of the deferent—is maximum at the limits and zero at the nodes for both planets. At the northern limit, the eastern side of the epicycle is raised further north for Venus and further south for Mercury — further away from the ecliptic in both cases. At the southern limit, the opposite is true: the eastern side of the epicycle of Venus is tilted towards the south, and that of Mercury towards the east.

1ô8 Ptolémèc, */'Almageste,* translation by G. J. Toomer, XIII, 2, p.599, H530,l.3-9. 169 Ptolémée, *'Almagestv,* translation by G. J. Toomer. XIII, 2, p.599, H530,l.3-9. 170 Ptolemy, *Almagest,* translation by G. l. *Too* . XIII, 2, p.599, H529,l.5-$.
171 Ptolemy, *Almagest,* translated by G. J. Toomer. XIII, 2, p.599, Hñ30,l.l2- t4.

towards the north, that is, towards the ecliptic in both cases."° For clarity, these different movements are summarized in the figures below.

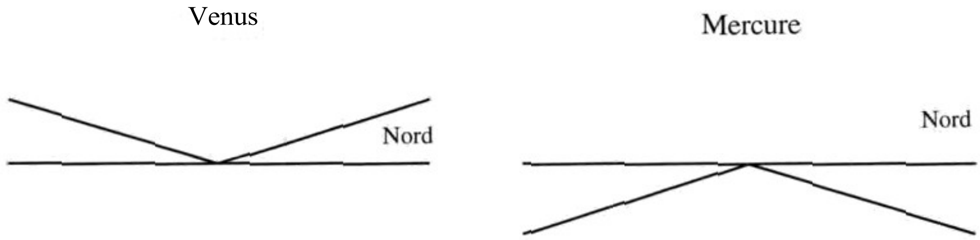

Fig. 3.13 at limits

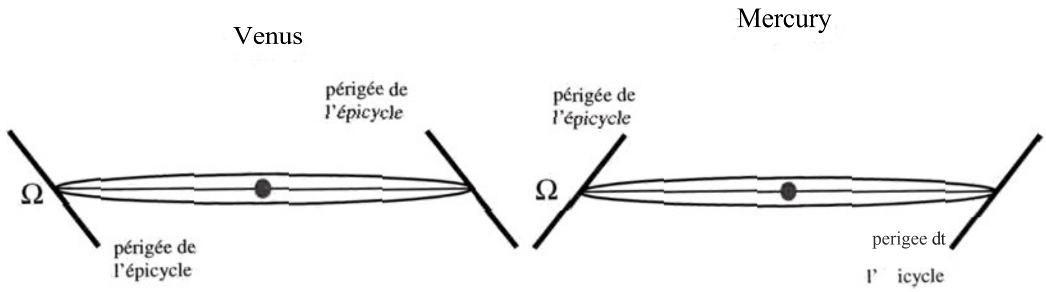

Fig. at nodes

The reasons why Ptolemy introduced these additional anomalies are unclear, especially since, as Olaf Pedersen points out, he could not obtain the data on which he based his calculations through direct observation, and it is difficult to know how he actually obtained them.[173]

As we have seen, nothing of the sort is present in Theon, whose model for the inferior planets seems to be the same as for the superior planets: a deferent fixed at an angle to the ecliptic. The table below summarizes the parameters associated with the two astronomers' models for the inferior planets. We can see that this time, Theon and Ptolemy agree on the maximum deviations in latitude that these bodies can reach.

172 Ro/dmde, */'X/mageste,* translation by G. 1. *Toomer,* X17I, 2, pp.499—600, Hññ0,I.)2-J 4-His 1,1.6. T73 O. Pe£lcrs¢n, A *Survey oflthe* ***Almagesl,*** yp.ñ7I -?72.

	VENUS		MERCURY	
	Angles	Maximum deviation in latitude corre undulating	Angles maximum	Maximum deviation in latitude corres ondant
In Ptolemy				
the inclination *of* the deferent on the ecliptic	0; 10' (north)	0; 10" (north)	0; 4S° (south)	0; 45 (south)
(ncfinaison de epicycle on the deferent following *the axis öe l"äepicycle* parallel to the deferent"	2; 30	$6^1/_3$°	1 6	‹°
Inclination of the epicycle on the deferent following the terra-center axis of l'é i cle	3; 30° "	2; 30"	7° 178	2; 30° '*
In Ptolemy *Book of Assumptions:* inclination of the deferent on the ecliptic	o; 10° 1^	0; 10°	0; 10" *	0; 10"
inclination of epicycle on the deferent following the axis of the epicycle parallel to the deferent	3; 30" "2		6; 30°	
In Théofi: Inclination of the deferent on the ecliptic t" ue"*	6 (constant)	g°	4 (constant)	4

174 Pioleniée, / 'Afriiagesie, translation by G. 1. Toemcr, XIII, 3, p.G0l, H534, 1.14-H5fi5, *I.S.*
175 Ptutenfc, *I'Ahnaf;e.die,* translation by G. J, T‹r›nwr, XIII, 3, p.60*, H53o. 1.4-H537, 1.4.
176 Pteilcnfie, / 'Alutageste, translation by G. J. To mar, XIII, 4, pp.G25-626, H.571, 1.9-H573, 1.8.
177 Pioléméc. *I'Alntagesie,* translation by Ci. J. Tt›omcr, XIII, 3, pp. GOD -602,H535, 1.5-1 3; XJII,4, p.f›25, H57 I ,

178 Ptolém e, *I'Alniazesie,* translation by G. J. Tooiner, XII I, 4, pp.62f,-627, H573, 1,9-Hfi73, 1.10.
179 Pioléjiiéc, f'ñfm«gesfe, translation etc. G, J. Tartly, GUI, 3, pp. fifll -602,H535, 1.5- 13; XIII,4, p.625, H57 l ,

I gO Pi‹ lñn\ v. /c• *Lii•i-e de Hj'pfuh?se,s.* \raduciinn *dv* I"arahc *dc* R. M‹›rclon, in *ifif)*DE/7. *) (I99ñj. j0 IR. I. I-4. I IU
Pr‹ n «, *j‹' /ii•* z' *Mrs l)›y›oihñfies,* translation by I"arahc dc R. Morc]on, in *VIDEO,* 21(I 091), y.?2. I.6-7.
I h2 Pft›l »cv. /r *Lii'i e* zf .Y Hi'jsn//th.sr.s, translation of the original by R. M‹›rclnn, in *MIDEO.* 2.1 (US), p.ñg. I.0-p.40.

1sà Pi‹›1ciiiéc. *le Livr‹• d‹.i Hypothèses,* translation of the urh by R Murelon, in *MIOEO,* 21 11993), p.34. 1.19-2?. 1fl4 Dclaiirc, p.1*9 11à5. 1.14-1 Il): the passage e‹›rrcstx›nJant cher Dupuis ssi peu cluir ïDupui,s, p.22? J.lt- 121, In fact, *as the* tables indicate, in *the Book of Hypotheses,* Ptolemy greatly simplifies these models, for reasons he does not indicate. For the superior planets, the inclination of the epicycle along its axis parallel to the deferent has become fixed. In the case of Saturn and Jupiter, this angle between the epicycle and the deferent is equal to the inclination of the deferent on the ecliptic, and its value is close to that attributed to the latter inclination in *the Almagest.* The plane of the epicycle is therefore parallel to that of the ecliptic, as Pedersen indicates." However, in the case of Mars, if we are to believe the version that has come down to us, this is not the case, contrary to what Pedersen asserts. With regard to the inferior planets, Ptolemy no longer mentions an inclination along the Earth-center axis of the epicycle, and gives a fixed inclination to the epicycle along its axis parallel to the deferent. He attributes the same inclination to the deferents of both planets, thus making the inclination of their epicycles solely responsible for the differences in their variations in latitude.

We can therefore see that the geometric models proposed by Ptolemy are both more complex and more diverse than those known to Theon. This is true of models designed to account for movements in longitude and those relating to variations in latitude. With regard to the former, Ptolemy's main innovations consist in the simultaneous rather than alternative use of the two hypotheses and in the introduction of the equant. The fact that he took the trouble to simplify his theory of latitudes in the *Book of Hypotheses,* compared to what he had proposed in *the Almagest,* suggests that, although he was convinced of their necessity, these complexities worried him, which, even more than his numerous statements to this effect, shows how important it was to him to account for phenomena.

184 **O. Pedersen,** ***A*** *Siirve) of the Almagesl,* p.392.

CHAPTER IV: ASTRONOMICAL PHENOMENA

In the previous chapter, we saw how Theon and Ptolemy intended to account for the motion of the planets. However, the concerns of these two astronomers—and of Hellenistic astronomy in general—did not end there. They addressed a number of problems that were clearly perceived as being of particular interest: eclipses, stations and retrogradations, the maximum elongations of Venus and Mercury, and the moments of first visibility of fixed stars and planets. Theon and Ptolemy do not deal with these issues in the same order. Theon returns to some of them in several places in his work. Ptolemy, on the other hand, addresses them in an order largely dictated by his presentation of the models themselves: he begins with the problem of the first and last visibilities of fixed stars, as he can address it as soon as he has set out his hypotheses concerning the sphere of the fixed stars; he then deals with eclipses after developing his model of the moon; then, once he has set out the models of the five planets, he addresses the problem of their stations and retrogradations,(6) and the maximum elongations of Venus and Mercury.(7) Finally, he addresses the question of the first and last visibilities of the five planets—which he can only do after developing

1 Ptolemy, f *'Almagesle,* translation by Ci. J. Toonier, VIII, ô, pp,4l 5-417.
2 **Ptolemy,** f'Afmagesfe, **translated by** G. J. Toomer, **VJIJ,** f -4, pp.371 -410; it is still necessary to deal with the problem **of simultaneous** risings, **culminations, and** settings of stars in chapter 5.
3 Ptolemy, *Almagest,* translated by G. J. Toomer, **VI, pp. 275-320.**
4 Ptolemy, ***Almagest,*** translated by G. J. Toomer, N & V, pp. 173-273.
5 **Ptolemy,** f ***'Almagesle,*** translated by G. J. Toomer, lX - Xl, pp.419-5fi4. fi
Ptolemy, *l'Almagesie,* translated by G. J. Toomer, Xfl, l -8,pp.555-588.
7 Ptolemy, *Almagest,* translated by G. J. Toomer, XII, 9-10, pp.587-'i96. S
Ptolemy, Almagest, translated by G. J. Toomer, XJJJ, 7-10, pp.636-647.

his theory of latitudes.[9] We are not obliged to follow this order here since we have already presented the models that account for the movement of the planets — moreover, as we have said, Theon does not follow it. We will therefore first deal with eclipses, then stations and retrogradations, the maximum elongations of Venus and Mercury, before dealing simultaneously with sunsets and sunrises — first and last visibilities — of fixed stars and planets.

I - Eclipses

In the time of Theon and Ptolemy, the explanation of solar and lunar eclipses had long been known. The whole problem was how to predict them. This is what Ptolemy set out to do, while Theon, due to the purely didactic nature of his work, limited himself to explaining the phenomenon, emphasizing its lack of regularity.

Theon begins by briefly explaining the principle of occultations in general terms: he *explains that if* we consider two stars, the one closest to Earth occults the other when it is directly below it, because our vision travels in a straight line. This explains solar eclipses, as the moon is closer to the Earth than the sun. Theon then explains lunar eclipses: the moon disappears when it enters the Earth's shadow, because it is opposite the sun. The rest of Theon's explanation on the subject consists of explaining why these two phenomena do not occur every time the two luminaries are in conjunction or opposition, i.e., once a month. He explains

9 Ptolemy, *f'Almagesle,* translated by G. J. Toomer, XfIJ, 1-6, pp.597-63fî. For fa Junr, he develops the question of variations in the latitudes of this celestial body in the context of his theory of the moon, in books IV and V, and not in book XIII.
10 Theon of Smyrna, *Exposition,* translation by *1.* Dupuis, pp.311-3 lS: Theon of Smyrna, *Exposition,* translated by J. Delattre, pp. 191-192.
1 J Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.31 S; Theon of Smyrna, *Mposiiion,* translation by J. Delattre, p.192.

as well as the reason for this being that the orbits of the two luminaries are not located in the same plane. Indeed, as we saw in the previous chapter, Theon attributes to the circle(s)[3] of the sun an inclination of half a degree relative to the ecliptic, and that of the moon an inclination of 5º or 6'. Not having to make precise calculations, Theon is not forced to choose between these two values. Ptolemy, on the other hand, had to do so, and chose the value of 5" put forward by Hipparchus.

Furthermore, since he attributes an inclination to the sun's orb, Théoii is forced to define the nodes differently from Ptolemy. Indeed, since the latter only considered the inclination of the lunar orb to account for eclipses, he defined the nodes as we saw in the previous chapter: These *are the* points of intersection of the inclined circle of June with the plane of the ecliptic. Theon, on the other hand, defines them as the intersections of the orbs of the moon and the sun. He specifies that this intersection passes through the centers of the two circles. Despite this difference, the 'nodes' have the same role in the theory of eclipses in Theon and Ptolemy: they *are* the places near which the conjunction must take place in order for the luminaries to have latitudes close enough for an eclipse to occur. It is implicit in Theon that the closer the two stars are to the nodes, the closer their latitudes are, and

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The greater the eclipse will be, until it is total if the conjunction takes place exactly at one of the nodes.

This being sufficient to explain why solar eclipses do not occur every month—and, incidentally, why some are total and others only partial—Theon sets out to explain the same phenomenon for lunar eclipses. He begins by explaining in detail, with figures, that if a luminous sphere illuminates another sphere, the shape of the latter's shadow depends on their relative sizes: if they are the same size, the shadow is a cylinder; if

12 Thêon Je 8mymc, *Exfi›osition,* translated by J. Dupuis, y.1 I ñ; Théon dc Smyme, *Exposition,* Translation of 1. Dclanre, p. 192.
I ñ or circles, depending o n the hypothesis adopted.
I4 Th5on dc Smymc, *Mposilion,* translation by /. Oupuis, p.û I ñ; Thdon de Smymc, *Exposition,* translation by 1. Delattrc, pp.192- I91.
IS Thdon dc Smyrnc, *Wposifion,* translation by *1. Dupuis,* m.1 I1-.33 I; Th/on de Smyrne, *Exfiiosîtion,* translated by 1. Delattrc, pp. IV2-I 96.

They are different sizes. The shadow is a cone. [6]However, the latter is not positioned in the same way depending on which of the two spheres is larger. If the luminous sphere is smaller, the shadow cone is such that its truncated apex is on its side, and the base widens indefinitely behind the other sphere. This is what Theon calls an 'indefinite truncated cone'. If the luminous sphere is the larger, the cone is such that its base is at the level of this sphere, and its apex behind the other: the shape of the shadow cone is 'limited'."

le cône

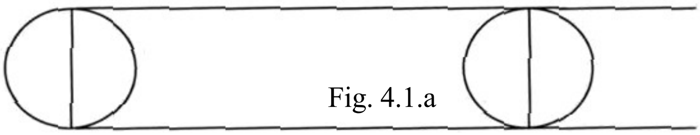


Fig. 4.1.a

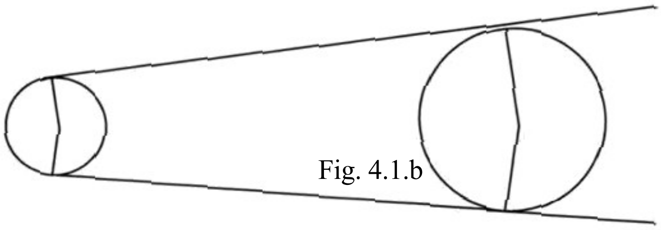


Fig. 4.1.b

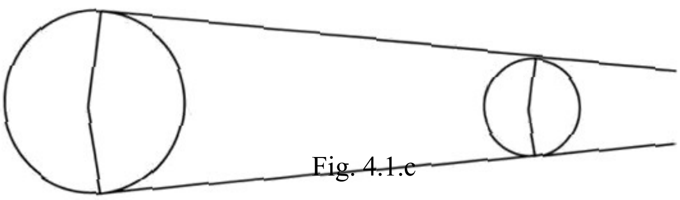


Fig. 4.1.c

1f› Thëon of Srr›yrne, *Exposition,* translation by). Dupuis, pp.315-.319; Théon of Smyrna, *ExPä8 Ation,* translation by J. Dclattre, pp.193- 196.
~~17'Thëon of Smyrna, *Exposition,* translation by J. Dupuis p.317.~~ 18
Theon of Smyrna, *Exposition,* translation by J. Dupuis p.317.

Theon bases his argument on Hipparchus' calculations to assert that this third scenario accounts for lunar eclipses. He mentions three facts that he considers together, drawing conclusions without specifying which fact is responsible for which. First, the sun is much larger than the earth (180 times larger according to Hipparchus); this implies that we are in the third situation and that the earth's shadow i s shaped like a finite cone. Second, the moon is much smaller than the earth (27 times smaller, again according to Hipparchus). Thirdly, the sun is much farther from the earth than the moon is (this time, Theon gives neither figures nor author). These last two facts, taken together, prove that the moon can be completely obscured by the shadow of the earth. Indeed, for the moon to be completely eclipsed, its diameter must be less than that of the section of the cone at the distance from the Earth at which it (the moon) is located. This depends, of course, on its diameter: if it were as large as the Earth, and even more so if it were larger, it would necessarily extend beyond the cone of shadow and could not be completely eclipsed by it. It also depends on the size of the section of the cone at the distance in question, and this size depends on two factors. Firstly, the shape of the cone itself, which depends on the relative sizes *of* the Earth and *the Sun* on one hand, and the distance between these two bodies on the other. Secondly, the size of the section of the cone depends on the position of the moon along the axis of the cone, i.e. on the distance between the Earth and the moon. Theon summarizes these two factors into one: the ratio of the distance between the Earth and the Sun to the distance between the Earth and the Moon. The higher this ratio, the greater the chance that the moon will be eclipsed, or even totally eclipsed, for a given ratio of the sizes of the three celestial bodies relative to each other, of course. This is illustrated by the figures below:

19 Thëon of Smyrna, *Exposition,* translated **by J. Oupuis,** p.319; ***Théon* of** Smyrna, *Exposition,* translated by J. Delattrc, **pp.**19**4**-195.

20 Theon of Smyrnc, *Exposition,* translated by *1.* Dupuis, p.319; Theon of *Smyrna, Exposition,* translated by *1. Del-uice,* pp.194- 95.

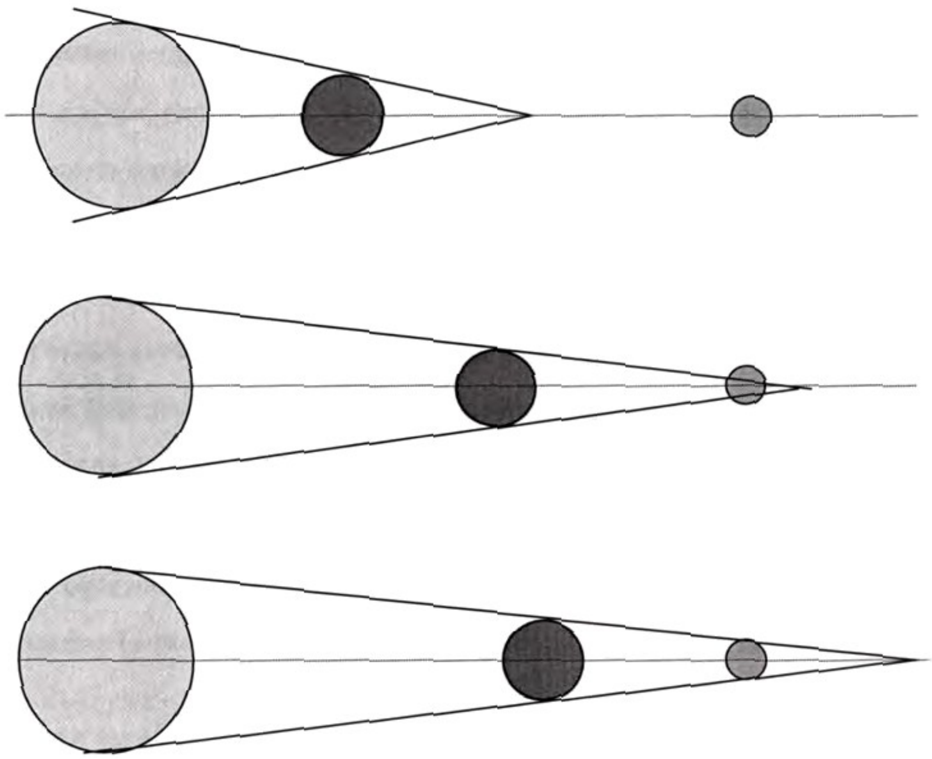


Fig.4.2

As we have said, Theon leaves implicit the respective roles of these last two factors—i.e., the size of the moon relative to the earth and the ratio of the distance between the earth and the sun to the distance between the earth and the moon. He simply states that the moon is smaller than the shadow and that, having no light of its own, it is thus totally eclipsed when the luminaries are each at a node during their opposition. He does not propose or refer to any calculation that would demonstrate that the ratio of the size of the moon to that of the Earth and the ratio of the distance between the Earth and the sun to the distance between the Earth and the moon are such that, together, they allow total lunar eclipses.

21 Theon of Smyrna, *Exposition,* translation by *I.* Dupuis, p.319; Theon of Smyrna, *Mposition,* Translation by J. Delattre, p.195.

Theon attributes the fact that there is no eclipse in June to *the fact* that the luminaries are in opposition to what they are not usually at the nodes when this event occurs.[2] It is clear that he also attributes partial eclipses to the fact that the luminaries are at a distance from the nodes that makes their latitudes sufficiently similar for there to be an eclipse, but not enough for it to be total.

Just like solar eclipses. However, Theon is not as explicit in this case, as he simply states that the sun, Earth, and moon must be exactly aligned for there to be a total eclipse, and that when their centers are only partially aligned, the eclipse is partial.° Nevertheless, this amounts to the same thing as defining the positions of the luminaries in relation to the nodes: it is only when the two celestial bodies are at the nodes that they can be aligned with the Earth, since the latter is located on the line joining the nodes—i.e., the intersection between the orbs passes through the Earth.

11 It goes without saying that Ptolemy deals with eclipses in a much more elaborate way than Theon. His aim is to obtain tables that will enable him to determine for which conjunctions and oppositions there will be an eclipse, at what time, how long the eclipse will last, and what its magnitude will be. (2)

When it comes to solar eclipses, Ptolemy had to take into account the fact that they can only be observed from certain places on Earth. A lunar eclipse, on the other hand, is caused by the shadow cast on the moon itself, and can therefore be observed regardless of the observer's position on Earth. However, to see a solar eclipse, one must be on the right side of the two celestial bodies. But the direction in which the moon appears—and to a lesser extent the sun—varies significantly depending on where one is on Earth: this is the phenomenon of parallax. Ptolemy therefore constructed a table to determine the parallaxes of the two luminaries and in

22 Theon of Smyrna. ***Exposition,*** translation by J. Dupuis, pp. 319-321; Theon of Smyrna, ü **yoff/Ïofï.** translation by J. Delattrc, p.195.
23 Theon of Smyrna, *M sitian,* ***vaduciion*** cix J. Dupuis. **p.319;** Theon of Sinymc, *Exposition,* translated **by J. DelalUe, pp.194- 195.**
24 Plolcmée, ***f'Almayesle,*** translation by **G. J. Toomer,** Vl, 7-10, pp.294-313.

latitude, and in longitude, regardless of their true position.° To this end, he calculates the average values of the distances to the moon—both at syzygies°' and at quadratures—2 ' and to the sun.°'

Ptolemy then developed tables to determine the times and positions of mean conjunctions and oppositions—i.e., conjunctions and oppositions of the mean sun and the mean moon—and to make the necessary corrections to obtain those of true *conjunctions* and oppositions—i.e., of the true sun and the true moon.[2] In fact, this could already be calculated from the tables of the movements of the sun and moon that he obtained by developing his models of the luminaries. But this would require calculating and comparing the positions of the two bodies around the appropriate period until the same position was obtained for both — whether we consider the mean or true positions. **This** would obviously be an enormous task, which is why Ptolemy undertook to provide a means of determining the exact times and positions directly.

Ptolemy then had to determine whether a precise conjunction or opposition would give rise to an eclipse. To do this, he calculated the "ecliptic limits" or "limit positions" of the sun and moon, i.e., the maximum distance at which they could be from the nodes for an eclipse to occur. The positions of the luminaries obtained from the previous tables can then be compared with these results.

For solar eclipses, he thus determines that, for latitudes between

between Meroe and Borysthenes — including Alexandria, of course — the moon appears tangent to the sun when the center of the true moon is located at 17; 41° from the nodes if it is north of the ecliptic, and 8; 22° if it is south. This corresponds to distances

25 He performs the demonstrations necessary for the elaboration of this fable in chapters 1 to 17 of Book V of *L'Aim‹ige.rte,* gives the table in chapter *18, and* explains its use in chapter J 9, also in Book V.
26 Oppositions or conjunctions.
27 Ptolemy, *f'Afmeges/e,* translation by G. J. ToumGr, V, 13, pp.247-251.
28 Ptolemy, *f'Almagesle,* translation by G. J. Toomer, V, 15, pp.255-257.
29 He explains how these tables were constructed in chapter 2 of book V I of *L'Almageslv,* provides them in chapter 3, and explains their use in chapter 4.
30 Ptolemy, The *Almagest,* VI, 5, translation by G. J. Toomer, pp. 282-287.

from the center of the average tune of 20; 41 and 11; 22" respectively. These results are only valid for *these* latitudes because Ptolemy must take into account parallax (see Fts 4.3 below), which varies with geographical latitude. The fact that the positions are not symmetrical with respect to the nodes, but differ depending on whether the moon is in the north or south, also stems from parallax: the
maximum parallax is not the same depending on whether *we* consider a parallax towards the north or towards the south.

parallaxe
en latitude

parallaxe
totale

moon

sun

ecliptic

Fig. 4.3

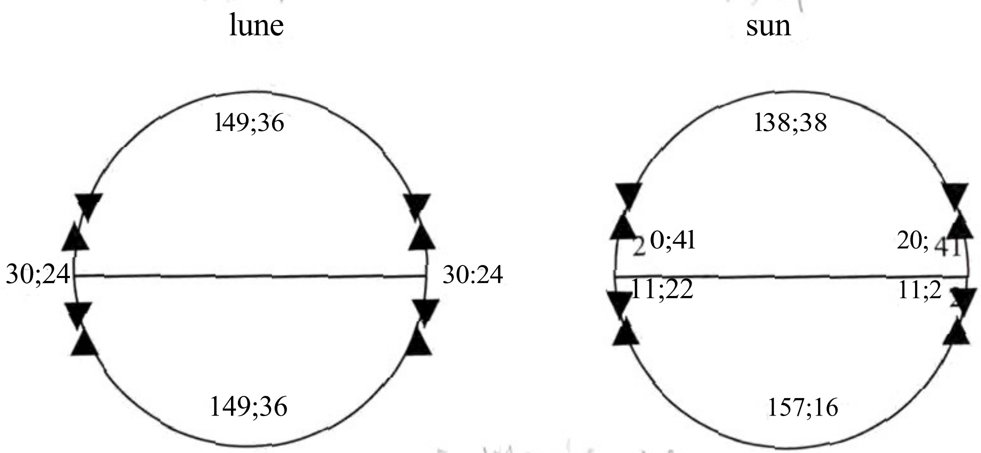
For lunar eclipses, Ptolemy calculates that the moon is tangent to the *Earth's* shadow—directly in front of the eclipse—when it is at 12; 12" from the nodes of its orbit. This corresponds to the positions of the mean moon at 15; 12º from the nodes.

The situation can therefore be summarized as follows:[35]

3 I Rolćmóe. / : ł /zønp«xre. VI.S, translation by G J. Toomcr, p 2b9.
32 black Rolćméc. f'. lJøtnge.see. VI, 5. translation by G J. Toomcr Fig. 6.1, p 2b'›.
33 Rolémóc. / .1finagesfe. VI.5. translation by G.J. Tnomer. pp.2łlñ-28a›.
s4 Rołćniée. ? : i/oMesie. VLâ. translation by G.7 Toomer. p.2s7.
3•i G.J Toomcr, *l•ioleinv'. Alni‹ipe.si.* Fig.H. p.2ßS.

eclipse de

eclipse de



pour latitude A travers

Fig. 4.4

position de la lune sur son orb

pour qu'elle soit tangeante à l'ombre de la terre / au soleil

11 It is of course impossible to determine the longitudes corresponding to these configurations, since syzygies—and other synodic phenomena—can occur at any latitude in the case of the moon. Rolémèe nevertheless seeks to further simplify the determination of eclipses by calculating the intervals at which they are likely to occur: how long can the syzygies that occur at a distance from the nodes less than those he has just determined be separated? The aim is to avoid calculating the positions of the luminaries at each syzygy. He thus determines that lunar eclipses can be separated by intervals of 5 or 6 months. The same applies to solar eclipses. However, solar eclipses can also occur at intervals of 7 months, which is impossible for lunar eclipses. ? longitudes ?

Ptolemy was then ready to develop tables that would make it possible to determine the magnitude and duration of a given eclipse based on the position of the moon on its inclined circle—measured from the northern limit, which could be found using

Afi What in the model of the moon developed by Ptolemy , and certainly by Hipparchus if, as we might think, he had already developed a model with an inclined circle (see ch. 3), is reflected in the fact that the nodes *are at rotation.*
37 Ptolemy, *Almagest,* VI, b, translation by G.J. Toomer, pp. 287–294.

tables."[8] Pour finir, Ptolemy even devotes three chapters to determining the orientation of the luminaries during the main phases of eclipses—the beginning and *end* of the eclipse for the sun and moon, as well as the end of immersion and the beginning of emersion for the moon. He thus provides a table for calculating the angle between the line joining the centers of the two celestial bodies and the ecliptic, as well as the angle between this same line and the horizon. ([9])

There is therefore little disagreement between Theon and Ptolemy on how eclipses should be accounted for: Theon simply attributes an inclination to the solar orb relative to the ecliptic, which leads him to define the nodes differently from Ptolemy. That aside, the difference between the two *astronomer is*, of course, that Ptolemy's work is incomparably *more* elaborate.

II - Stations and retrograde motion

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Another very important synodic phenomenon is the stations and retrograde motions of the planets: the fact that sometimes a planet can interrupt its

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apparent movement from west to east and move from east to west for a certain period of time.

period of time, before resuming its movement eastward.

Heading towards Oucst-Est

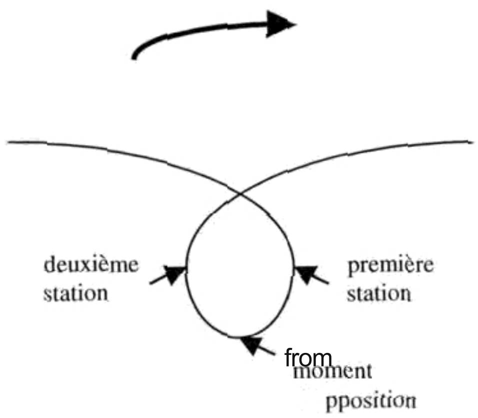

Fig. 4.5

38 He provides the corresponding demonstrations in chapter 7 of book VI of *the Almagest,* gives the tables in chapter 8, and explains their use in chapter 9 (for lunar eclipses) and 10 (for solar eclipses).
39 Ptolemy. *The Almagest,* VI. I f, 12, l fi, translation by G.J. Toomer. pp.3 l 3-320.

Ptolemy and Theon agree on the characteristics of this phenomenon: both describe it as specific to the five planets, with different durations and arcs for each, [40] and specify that it does not concern either the sun or the moon. [4] They also agree on the general way in which it should be explained. However, as we shall *see,* their explanations differ in detail.

In fact, although he goes on to address the geometric problem, Theon's initial explanation remains vague and somewhat confusing. He first presents the opinions of Plato and Adrastus, not only with regard to retrograde motion, but also with regard to the direction of planetary motion in general. He emphasizes that, for Plato, the planets actually move from west to east, whereas for Adrastus, this movement is only apparent. This is somewhat surprising, as one would rather expect retrograde motion to be described as an appearance, given its unusual nature. However, it is indeed the "normal" motion that Adrastus, unlike Plato, describes as an "appearance." The explanation for this paradox seems to lie in section XXI of the treatise. However, there is disagreement among translators as to *the meaning* of the passage in question: Duptiis treats it as referring only to retrograde movements, while Delattre considers it to apply to all planetary movements regardless of their direction. What *we* have just noted regarding Theon's opinion attributed to Adrastus concerning movements from west to east clearly supports this second interpretation. In this passage, *Thion* explains that planetary *movements* are only an appearance due to the fact that the planets, being below the sphere of the fixed stars, are seen with this

40 Theon of Smyrna, *Exposition,* translated by J. Dupuis, **p. 28;** Theon of Smyrna, *Exposition, **translated*** **by** Delattre, p. 170. In fact, Theon simply **states** that retrogradations **occur** differently for each planet, but it seems clear that he is referring to their duration and the length of their arc.

4l 'J'héon de Smyme, ***ExpOS y/on,*** translation by J. Dupuis **p.239;** Theon of Smyrna, ***Fxposilio** i,* translation by J. Delattre, p.l42.

last one in the background. This nevertheless remains open to interpretation — two interprétations, pour être exact.

Theon nevertheless undertakes to explain the phenomenon of retrogradations géométriquement.[43] Pour ce faire il raisonne à partir de la figure suivante:[44]

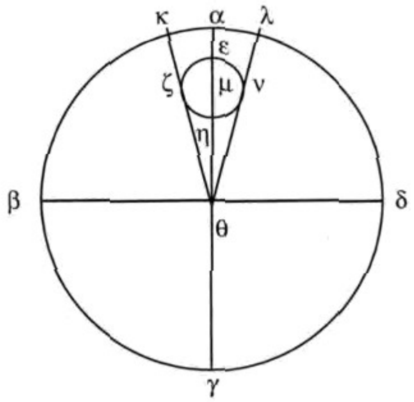

Fig. 4.6

42 There are two interpretations to be precise. First, we can consider that Theon's remark is intended to be very general, as is often the case in his work, and that appearances in question are not particularly misleading, since the fact that we see the planets moving backward relative to the sphere of the fixed stars is in no way incompatible with the idea that they are actually moving themselves in that direction. All the more so since it is in relation to the fixed stars that we define The problem with this interpretation is that it does not fit with Theon's insistence on opposing Adrastus' notion of appearance to Plato's notion of reality. The second interpretation is that the appearances in question are indeed deceptive in Adrastus' mind because the planets, instead of moving from west to east on their orbits (for example, in one year for the sun), they simply change direction relative to the movement of the sphere of *the fairies* from east to west. It should first be noted that this interpretation is not obvious, contrary to what Theon of Smyrna seems to think. *Explanation,* translation by J. Dupuis: he translates the term "cmxpoofhjœv" as "resistance," thus having Theon assert that it is "because of resistance" that a planet appears relative to the zodiacal zone above it (Theon of Smyrna Er z/ziœ. Translation *by* J. Dupuis, p. 241). In fact, as De Attre points out, this term also means that something is placed in front (Delattre, p. 143, note 49). In this case, which seems most likely given the immediate context in which the term appears, becomes neutral as to which of the two interpretations is correct. Furthermore, if Theon and Adrastus use the term ‹pœvztaoio is not *fviâenl that they* refer here to a deceptive appearance Note that the term can mean image and that refers to the fact that we project an image of the planets onto the celestial vault. Nevertheless, other elements seem to support this second interpretation. First, Thfon says of the two luminaries that they are , left behind (Thëoii of Smyrna, M,p«riiido, translation by J. Dupuis, p.239) oti That they "ÎresieniJ behind" (Theon of Smyrna, *Exposyion,* translation by J. Delattre, p.142). Secondly, as we have seen, Theon clearly contrasts Adrastus' opinion with that of Plato. Thirdly, further on, Theon explicitly considers the possibility that the cycle is simply left behind (Theon of Smyrna, *Exposyion, translation* by *J.* Dupuis, p. 309), although at this point he does not really take a position in favor of this possibility, whereas he clearly sides with Adrastus in the passage that concerns us. 43 Theon of Smyrna, *Exposition,* translation by J Dupuis, pp. 307-309; Theon of Smyrna, *Mposition,* translation by J. Delattre, pp. I89-190.

44 Theon *of* Smyrna *Expozifion,*

, traduction de J. Dupuis, p.306.

Théonentrepren d nevertheless explains the phenomenon of retrogradations

gÂo étriquement.[45] P To do this, he reasons on the basis of the following figure:[46]
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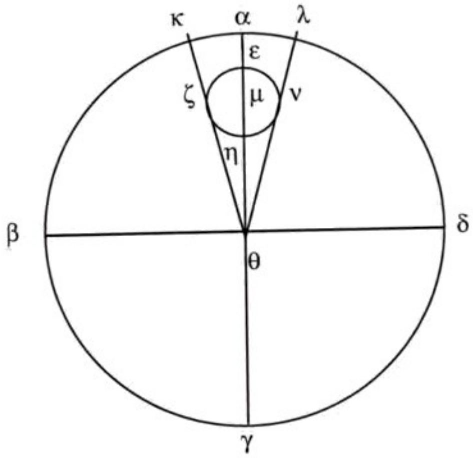


Fig. 4.7

Theonexplainsthusthat when the planet travels along the arcs Çe and cv on the epicycle, it appears to(move) (along) the arcs x‹x and W on the ecliptic. Theon explicitly states that this(constitutes) (a) non-retrograde motion: he says further down that after having traveled (t') s(cvqÇ, "being) (far) frompoint Ç, it [the celestial body] will advance again (47) orresume its forward motion." Appearing l'arc εν, la planète seems to slow downwhen it (approaches)v, which constitutes its first station. When it travels along the arc vpÇ, it will seem todescribethe arc kiyx. Theon specifies that this is indeed the retrograde motion

retrograde movement:

> ... as it approaches point v and begins to move away from it, it [1'star) will appear to stop again and finally retrograde [4 e]

> ⁰approachingfromN[v],and beginning to move away, it will appear to stop again and retrograde. ..50

45TheonfromSmyme,£xyos*ff'Dn,translationfromJ.Dupuis, pp.307-309; Theon of Smyrna, *Exposition,* translation by J. Delattre, pp.189-190.

46 Theon of Smyrna, *Mposilion,* translation by J. Dupuis, p.306.

47 Theon of Smyrna, *Exposition,* **translated** by J. Dupuis, p. 307. 4d

Theon of Smyrna, ***Exposition,*** translated by J. Delattre, p. 159.

49 **Theon** of Smyrna, *Exposition,* translated by J. Dupuis, pp. 307–309.

When the planet is close to (, it appears to move slowly, and finally seems to come to rest at x. (is therefore the point on the epicycle at which it is at its second station.

This description suggests two thoughts. First, the direction in which the movements occur seems to be reversed. We have already noticed this by following the description in the figure. Curiously, this is not simply a reversal of the latter in relation to the usual meaning—representing movement from west to east in a counterclockwise direction—which, after all, is little more than a convention. Theon clearly states that when the plane moves along vr|Ç and appears to describe the arc Ztxx, the movement is "towards the following signs of the zodiac" or "towards the signs of the zodiac that follow." — eu zœ enopcrn cmv Çtn6t v —, i.e. from west to east, when in fact it is a retrograde movement! We cannot assume that Theon, unlike Ptolemy, actually meant "from east to west" by this expression. Indeed, he specified above that the movement of the stars in longitude is such that "they go to the signs that follow them and not to the signs that precede them"*' — "eu [...] zœ exopcvœ xm Çm6iciv om ètè xn npoij ,upcvn ." Similarly, Theon specifies that when a celestial body moves from Ç to e, this movement, which is not supposed to be retrograde, takes place "towards the preceding signs of the zodiac" or "to the preceding signs" — "zporJ o vtx",' therefore from east to west. This is so curious that it is tempting to attribute it to transmission errors.

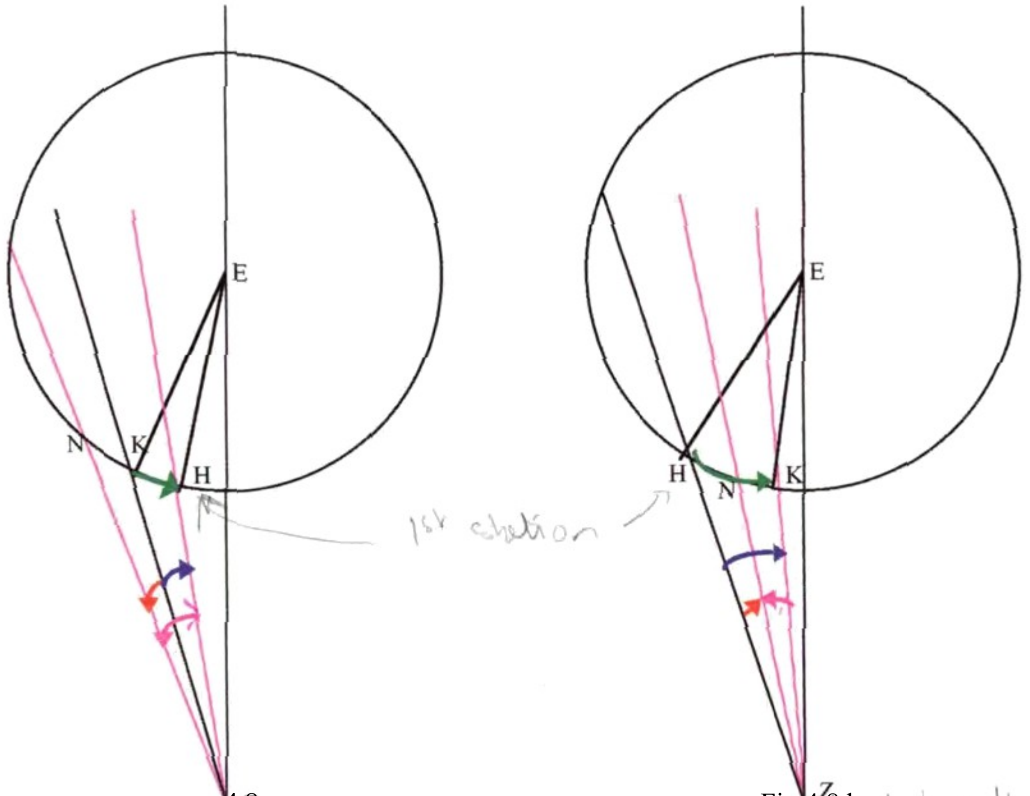
Secondly, Theon fails to take into account the movement of the epicycle on the deferent. He does explain that this means that stations and retrogradations can occur anywhere on the ecliptic for the same planet.

50 Theon of Smyrna, *Mposition,* translation by J. Delattre, p.1 89.
51 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.307.
52 Theon of Smyrna, *Exposition,* translation by J. Delattre, p.189.
53 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.221.; see also translation by DelatireF .128: "Indeed, it is toward the signs of the zodiac that follow them, and not toward those that precede them, that they move in their own progression, carried in the opposite direction of the whole, in a movement that is said to be according to their own *'longitude'..."*
54 Theon of Smyrna, *Mposition,* translation by J. Dupuis, p.220, t.17-18.
55 Theon of Sinyrne, *Mposition,* translation by J. Dupuis, p.307.
56 Theon of Smyrna, fxyor*rf£1ti, translation by J. Delattre, p.189.
57 Theon of Smyrna, ExyosîifGZi, translation by J. Dupuis, p.306, 1.13.

But he ignores the consequences for the position of the two stations on the epicycle, as we have seen. Theon places the stations at the points of the epicycle of the lines passing through the Earth. This is the main difference between what Theon and Ptolemy propose.

mous ement non rétro rade

real retrograde motion

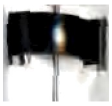


4.8

Fig.4.8 b

Fig. .a

Ptolemy places the station points not at the points of tanJnence, but at the intersection of the epicycle with a line passing through the Earth (here at Z), in order to take into account the displacement of the epicycle on the deferent. The figures above



show what happens shortly before (Fig. 4.8 a.) and shortly after (£ig. 4.8 b) the planet reaches the first station, represented by H.

Consider Fig. 4.8 a., which shows the planet moving from a point K located i west of H— i.e., on the apogee side relative to the first station. Ptolemy demonstrates that in this case, the ratio of angles HZFi to Kl- 11 is less than the ratio of the planet's speed on the epicycle to the speed of the epicycle on the deferent

He then chooses a point N such that the ratio of the angles ' a lx El I is equal to the ratio of the speed of the planet on the epicycle to the speed of the epicycle on the deferent:

/ f F l f= ro ' cii

This implies that the planet travels through angle i:l (— from east to west — at the same time as the epicycle moves from west to east on the deferent by an angle equal to ' "

Seen from Earth, the planet appears to move through an angle KZH," while the center of the epicycle moves through the angle : .'"" .. being greater than KZH — which Ptolemy previously demonstrated — the planet appears to have moved by the angle i Z and this from west to east, therefore by a non-retrograde motion.

Let us now consider what happens when the planet has passed the first station on the epicycle (Fig. 4.8.b): "relative to H, point K is now on the side of the epicycle's perihelion."

Ptolemy shows that the ratio of angles HZE to UL II is now greater than the ratio of the planet's speed on the epicycle to the speed of the epicycle on the deferent:

HZk / III- l> i". / ‹o

i8 The fact that it **is represented** here so close **to the perigee of** the epicycle i s solely due to reasons of **clarity. dc** nianiire **to jxiusoir tcprcscntcr** the arcs;i su left ctairemcni

J'7 Rolcnic•a. :/ f/o/ rr/‹•, XU. Translation by G J Toomcr. pp.559-561.

(III We have **not reproduced** the **entire** inscription in order to remain closer to **the figures of** Ptolemy, who does not do so either

or at least would appear to shift from angle h.TH if **it** were the only side to **be** obscin'c diroctCmcnt.

ri2 **Rolcm‹ir.** / : i//rg«.rze. XII. 1, translation by G.J. Tooiuer. pp.5f›1-ü52

L›? Here we no longer follow the nomenclature **used by Rolégiéc ¡x›ur on points** H. k **AND** N. **in** order to **obtain a more** precise **designation** of the position of ***the rggte, Mig 4.8,o.***

Ptolemy is not as explicit as in the first demonstration, but the situation can be summarized as follows: "There is a point N such that the planet travels through the angle *l II-Y* in the same amount of time that the epicycle travels from west to east through the angle ï.
The planet therefore appears to move through the angle HZK while the epicycle appears to move through I:.' As the latter is less than HZK, the planet appears to move through *the angle I!Z.ü,* from east to west, i.e. in a retrograde motion."

However, we cannot speak of a contribution by Ptolemy in this regard, at least as far as the positioning of the station closer to the perigee than the point of tangency is concerned. He himself attributes a lemma that tt uses to Apollonius (second half of the 1st century BC), "woman who refers to a position of the planet at the first station." He even goes so far as to demonstrate the equivalence of this relationship with another, corresponding to the eccentric hypothesis for superior planets, "to then demonstrate how the phenomena of retrogradations can be explained in such an eccentric hypothesis, *which* in fact he himself does not defend, and which is of little help to him in the rest of his study. One may therefore wonder why Theon does not take into account the movement of the epicycle. All the more so since it is difficult to see how such a model could account for the phenomena For the superior planets in particular, due to the reasonable size of their epicycle relative to the radius of their deferent, the tangents to these epicycles separate them into two arcs whose lengths are fairly close; the planets concerned would then spend nearly half their time in retrograde motion. Consequently, although it remains possible that Theon simply

64 Still concerned about paralleling it with the previous demonstration. Gfi Rolémée.
l:t lmageste. Xll. 1. Translation by G.J. Tooi'icr. pp êtr l-fi62.
66› Prot›ablenicnt began in the year 249 and lasted until the end of the third century. In fact, according to Eutocius, he was active during the reign of Ptolemy Euergcics f24f›-221) (Eutocius, 6'o›iiiiicntori' on *.4pr//oniii.s* '€i›nies, Apoll. Perg. ed. Helbcrg ii. 165. i-l7il. 26). It is likely that Rolcmacus Chennus refers to him as having worked during the reign of Ptolemy Philator (221-2tl4) (apud Photii DU.. ced. *csc.,* ed. Bekker l S1 b 1d): ïn , *?rle•ciions illiisirolinff the* ffJ.sforv *‹if C ‹•I'*
.t laihe»ioiic.s. Vol.11. pp.27G-277.
f›7 Designed as here, point H as an intersection point c•ntre l'cpici cle ci a straight line passing through the earth that intersects the epicycle at two points. i.c. not tangent to the cpies cle
Rolèmcc. *:l Iiwazeste,* XII. I. translation by G.J. Toomer, pp.559-5 fiG
68 Toonier specifies that this relationship is also valid for the inner planets, if we give the center of the excentric a rotational speed that is not equal to that of the mean Sun, but to the
sommr dc reile dernière et de l'anomalie de la plan‹its. Toonicr. Rolenn 's Atmagest. p.555. noie 2.

ignored the work of Apollonius, it is undoubtedly more likely that he neglected the movement of the epicycle for the sake of simplicity, not wanting to go into detail in a work whose main purpose was to provide a general and approximate understanding of the phenomena.

It should also be noted that Ptolemy does not stop at explaining the phenomenon we have just described. His aim is to calculate the elements *that are* known to vary from one retrograde motion to another: the angular value of the retrograde arcs and the duration of a retrograde motion—between the first and second stations. To do this, he developed tables indicating the *position* on the epicycle of the first and second stations (counted from the apogee of the epicycle).(6)These results are expressed as a function of the position of the epicycle on the deferent, counted from the apogee of the latter. The reason for this is that he attributes the differences in length and duration of retrogradations for the same planet to the eccentricity of its deferent, which has the effect of varying the distance of the epicycle from the earth. A model comprising an epicycle on a concentric deferent — or its eccentric equivalent, with a mobile eccentric
— can certainly explain the phenomenon of retrogradations itself, as we have seen, since this is Theon's model. But it is not sufficient to explain the variations in the arcs and durations of retrogressions for the same planet. Theon makes no mention of this fact, nor does he propose any hypothesis that could explain it.

In order to draw these tables, Ptolemy performs his calculations for three positions of the epicycle on the deferent at the moment of opposition (in the middle of the arc of retrograde motion): at apogee, at perigee, and at the position of mean distance, i.e., equal *to* the radius of the deferent.[7] In fact, the demonstration is similar to what was already presented for Fig. 4.8,b; the situation is such that point K is now at the epicycle's rigée to represent the situation a t opposition (see Fig. 4.9 below).

below).

69 Ptolemy, *Almagest,* XII, 6 & 7, translation by G.J. Toomer, pp. 583-58d.
70 And this for each of the five planets, of course. Ptolemy, *Almagest,* XII, 2-f›, translation by G.J. Toorner, pp.562-58.3.

For each planet and each position of the epicycle, Ptolemy first determines the angle between the first station (Z) and the middle of the retrograde arc (the perigee of the epicycle, H, where the planet is located at the moment of opposition) as seen from the center of the epicycle (A), i.e., ñ.-tl4. Then he calculates the angle subtended by the corresponding arc, ZH, as seen from Earth (G): ZGH. He obtains the angle traversed by the epicycle on the deferent in the same time, (i lc il .), taking the displacement in longitude corresponding to the anomalistic motion he has already calculated (d!-\H) (the ratio ‹»r/ is of course known for each planet). 11 thus obtains the angle between the first station and the perigee, PGF., equal to ZGH.

- 11(l i. Since the year of retrograde motion is symmetrical with respect to the pürigée dc l'épieycle, which marks the opposition, it is simply twice C€i1› Ptolémûe also specifies how long the retrograde motion lasts — which can be obtained directly by dividing the angle of retrograde motion by the speed of the planet in anomaly, mi .

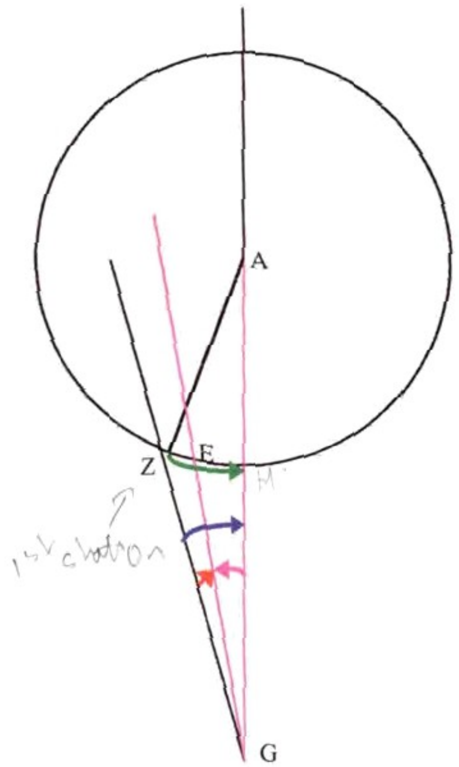


Fig. 4.9

Using these results, Ptolemy constructed a table by interpolation indicating, for each planet, the position of the two station points on the epicycle for any *position* of the latter on the deferent. [72] It is thus possible to obtain the value of the arc of retrograde motion—the difference between the positions at the two stations—and, from this, the duration of this retrograde motion for any position of the epicycle. For Ptolemy, accounting for retrograde motions in this way was a means of demonstrating the value of the models he had developed in previous books. "

III - Elong Maximum retrograde positions of Venus and Mercury

Rolémée addresses the problem of determining the maximum elongations of the inferior planets immediately after that of retrograde motion. Theon does not deal with the question of maximum elongations in detail, but simply gives figures: 20' maximum elongation for Mercury and 50º for Venus, whether these stars are west of the sun—visible in the morning—or east—visible in the evening. [74] Nevertheless we can reconstruct his conception of these elongations based on his hypothesis that Mercury and Venus revolve around the Sun and that their orbits have the same center as *that of* the luminary [5] '

71 counted from the apogee of the epicycle.

72 Ptolemy explains its construction in chapter 7 of book XII, and presents it in chapter 8, p. 588. The positions of the epicycle are given at intervals of 6".

73 Even if, as Pedersen notes, he cannot do so directly (Cl. Pedersen, *A Survey of the Almagest,* p. 330 & p. 354).

74Theon of Smyrna, *Exposition,* translation by *1.* Dupuis, p.303; Theon of Stnytne, *Exposition,* translation by J. De tre . 185.

75 Tliéon de Srftytne, *Exposition,* translated by J. Dupuis, p. 30; Théon de Smyrne, *Fxpositiun,* translated by J. Delattre, p. 154.

. lat , p

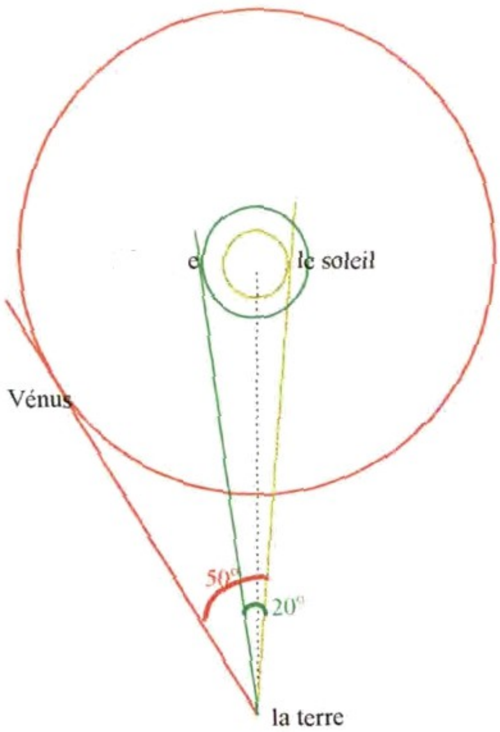


Fig. 4.10

As we have seen, the hypothesis of Mercury and *Venus* rotating around the sun is not the only one considered by Theon regarding the configuration of these celestial bodies, but it is nevertheless the one he prefers. The resulting conception of elongations is similar to that of the heliocentric system, precisely because Venus and Mercury revolve around the Sun according to both theories, and their movement around the Earth with the luminary desired by Theon does not influence the elongations.

The other hypothesis proposed by Theon is closer to that of Ptolemy. It states that the centers of the epicycles of the three astrea are aligned—those of Venus and Mercury thus pointing toward the mean sun. It can be represented as follows

76 Thëon of Sp *Exposition,* trWnction dr:: *I.* Dupuis, *p* 313: Theon 'the Sms

77 Tldott dc Strzyme, rpzsifion, translation ¢lg / DoçXzîs, p,30î; Thdoa de Sztgmie, Wj›osifiozr, translation by / Delattzc, /gs.184-185.

Tne, *Exposition,*

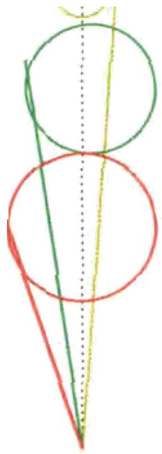


Fig.4.11.a

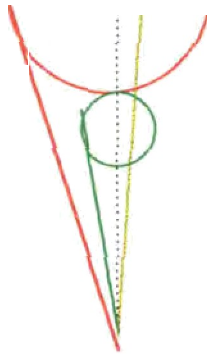


Fig.4.11.b

11 It is interesting to see whether Ptolemy, based on his models, arrives at different values. We will not go into the details *of* his *demonstrations*, but will simply summarize his approach. The aim is to determine the maximum angular distance that can separate the true positions of Venus and Mercury from the true sun.[7] From this, we can see that his models are off-center and thus alter the distance of the epicycle from the earth, and this alteration varies depending on the position *of* the epicycle on the deferent. Furthermore, for a given position of the epicycle, one cannot simply take as the maximum elongation the angle subtended by the center of the epicycle, the earth, and the points tangent to the epicycle on the line passing through the earth —

i.e. the points on the epicurean circle that Theon considers to be the two stations:

Fig.4.12

78 Rolcinéc. / 't/zii‹iu‹ .s/‹ . translation by G. J. Tootucr. X[1. '?. p.587.
The fact that Theon also alludes to a x ü lorigotions s raics ‹kms astres plittöt ctiic tnoi crimes csi c› idcnt according to the context in which he mentions it.

There are several reasons for this. First, what we are looking for is the elongation relative to the true sun, whereas the direction of the center of the epicycle is coupled to that of the mean sun. Second, the center of the epicycle is in the direction of the mean sun *as seen from Ju ceiilrv Jv l'éqncmt,* not from *ic•rre.*

Ptolemy performs the calculations not only for each of the two planets, but also distinguishing between cases where the planet is an evening star, with an eastern elongation relative to the sun, and cases where it appears in the morning, being west of the luminary.

The way in which Ptolemy proceeds to obtain the maximum elongation in the case of Venus, when it is west[79] of the Sun—i.e., when it appears in the morning—can be summarized as follows

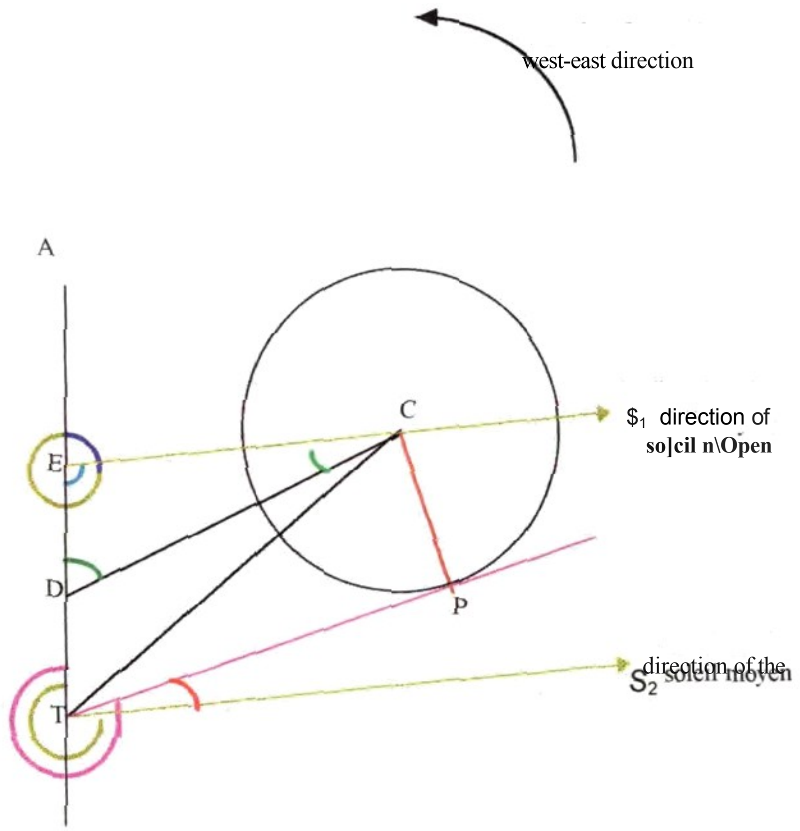


Fig 4 .13

7'2 Rolcméc. ' .1/Ariège.src. translation by G . J. Toomer, XII, 9, pp.588-591.

First, we must determine the maximum elongation relative to the mean sun for given positions of the epicycle on the deferent. In fact, Ptolemy gives himself, rather than the positions of the center of the epicycle, those of the planet: "he will establish a table of maximum elongations for the positions of Venus and Mercury at the beginning of each sign of the zodiac."

Unfortunately, although the maximum elongation is not equal to the CTP angle, it nevertheless corresponds to a position P of the planet on the epicycle at the point of tangency of the line passing through the Earth T. Now we must take into account the fact that it is from the center of the equant and Con of the Earth that the center of the epicycle appeared in Ía.

, direction of the mean sun. To do this, we must determine i' I This is equal to the difference between the angle '. ! i' and the angle - ," where A represents the apogee of the planet's deferent. Rolémée obtains **i** very easily since he has given the position of the star " And !' is equal to the angle ' ," which is of course equal to 360" -

' the angle DES " The latter is equal to l (l7 LEt'. This gives us 1* l s . maximum elongation relative to the mean sun. Obtaining elongation relative to the true sun is now straightforward: simply add (or subtract) the corresponding anomaly equation.

In fact, Ptolemy does not represent either the angle f' F . or the direction S of the mean sun as seen from Earth. He simply calculates the angle '• . and it goes without saying for him that this is the position of the mean sun — from 11 he deduces the position of the true sun. We have indicated the other angles only for clarity.

8U Ptolûincc. f '4 lampe.ne. XII. 'J. translation by G.J. Toonicr. p.sg7. 8 l
Piolcir«ic. ? .4/«weiie. XJI.)ti. induction by CI.J 3'ooiuer. p.S'il› b2
both Sppëricuo 5 18tl",
83 For the demonstration he gives as an example, Rolëméc chose to place it in 'T i›. ù
I'‹iquîno.xc of spring: Ptoism. ' i»• ••i•. xi1. UN. translation by G J. Toomcr. p.5ä8.
h4 higher than l8O" 85
iiifcricur ä 1 bai". *8ï›*
In *citct:*
A E S = l tltl" - i.'I:‹
The sum of the angles of a triangle is l8ti*:
f›l 1 * i ' f '1 ›+ I\ l 3t'= 181)*

donc:
AEC = 180° - [180° - (ECD + EDC)] = ECD + EDC
Ptolémée utilise assez fréquement cette propriété.

Ptolemy then performs a similar demonstration, this time placing the planet east of the sun:

E

D

T

Fig.

11 then calculates the same maximum western and eastern elongations for Mercury, adapting his demonstrations to the model he developed for this planet. In fact, in the case of Mercury, he cannot proceed in the same way because he cannot calculate the planet's mean longitude from its true longitude, due to the rotation of the center of the deferent. Consequently, he does not consider the planet directly at a given longitude; he chooses a mean longitude which implies for Mercury a position close to the beginning of a sign for its maximum elongation. Since this true longitude is only close to the longitude that interests him, he must consider two such cases, corresponding to the true positions of Mercury located on either side of the true longitude of Mercury.

87 Piolémfe, *l Slmagesie,* translation by G. J. Toomer, XII, 9, pp.590-591.
88 Ptolemy, *Almagest,* translated by G. J. Toomer, XII, 9, pp. 59 l -595.
89 At the beginning of Scorpio, 11} 0°, for eastern elongation, and at the beginning of Taurus, @ 0º, for western elongation.

of the other from this beginning of the sign. li then takes for the elongation at this precise longitude the value obtained by interpolation, which can be represented as follows:

$$\eta-\eta_{1} \;\_\; \lambda-\lambda_{1}$$

$$\Leftrightarrow \quad \eta=\left(\frac{\lambda-\lambda_{1}}{\Delta\lambda}\times\Delta\eta\right)+\eta_{1}$$

where:

Z: longitude of the beginning of the sign
k: longitude of the first position of Mercury considered (west of Z) b2:
longitude of the second position of Mercury considered (east of Z)
rJ: maximum elongation when the planet is at k.
r}: elongation when the planet is in Z, ijt:
elongation when the planet is in 2

Such calculations enable him to obtain the table of maximum eastern and western elongations for the two planets when they are located at the beginning of a sign. Curiously, Ptolemy does not indicate a method for obtaining the maximum elongation between these positions.

It should also be noted that, for a given longitude of the planet, the eastern and western elongations sometimes vary considerably—up to 11.02° for Mercury when it is in Q 0º. As we have seen, the figures given by Theon are the same for eastern and western elongations. However, this apparent lack of symmetry in Ptolemy's table is undoubtedly attributable to the fact that the two elongations are given for the same position in longitude of the planet and not of the epicycle. It is therefore certainly preferable not to attribute too much

90 Ptolemy, *Almagest,* translation by G. J. Toomer, XII, 10, p.596.
9 l In fact, its maximum elongation in the west, as the "morning star," is 16; 35º, and its eastern elongation is 27; 37º. Ptolemy, *Almagest,* translation by G. J. Toomer, XII, 10, p.596.

importance to this apparent difference between the two authors. Especially since, when we consider the maximum value of Mercury's western elongations compared to its eastern elongations, the difference is small: 28; 37º (9) ' and 27; 37º " respectively, a difference of only 1°. The same applies to Venus: 46; 47º '(4) and 47; 34º " respectively, a difference of 47'.

More unusual is the fact that the figures put forward by Theon do not match those of Ptolemy: 20º of elongation at most for Mercury, and 50º for Venus, as we have seen. The mere fact that these values represent tens already suggests that they are intended to be approximate. This explains the difference from Ptolemy's calculation for Venus. Indeed, as we have just seen, the highest maximum elongation he obtains is 47; 34º '(7) ›which is only a difference of 2; 26º from the value put forward by Theon. Moreover, this figure remains lower than that of Theon, which, given its imprecision, can be assumed to be a maximum value. The same cannot be said for Mercury. The difference between the highest value determined by Ptolemy, 28; 37º '(8) ›and that given by Théoii i s 8; 37º, which is certainly not negligible—especially compared to 20º or 30º. Moreover, this time Ptolemy's value is higher than Théon's. Mercury certainly has a reputation for being difficult to observe, precisely because of its low elongation. Nevertheless, 8º represents a considerable angular distance, and the difference between the two authors is therefore somewhat surprising. Since Mercury's period of revolution—around the sun in the heliocentric system and on its epicycle for Theon and Ptolemy—is only about three months, this difference would suggest irregular observational habits on the part of one of these astronomers.

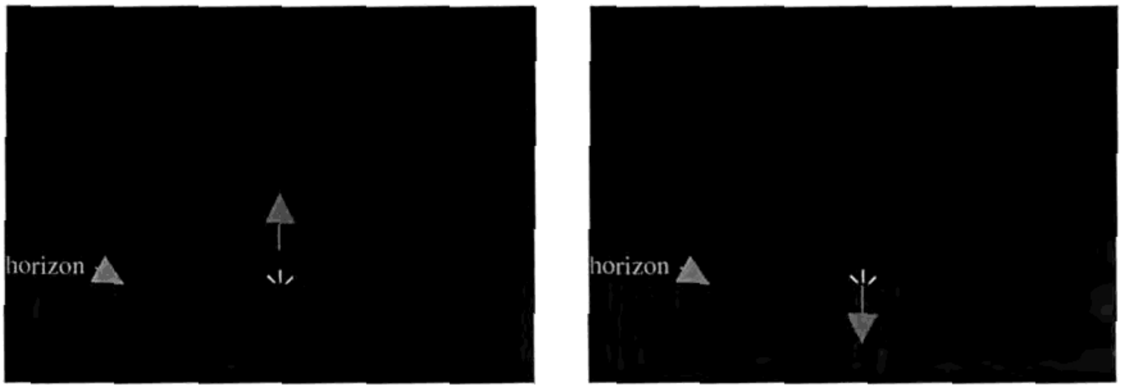
92 In 0º. Ptolemy, *Almagest,* translation *by* G. *1. Toomor,* XII, *10,* p.596.
93 In Q 0º. Ptolemy, *Almagest,* translation by G. J. Toomer, XII, 10, p.596.
94 In 0º. Ptolemy, *Almagest,* translation by G. J. Toomer, XII, 10, p.596.
95 In ri 0º. Ptolemy, *Almagest,* translation by G. J. Toomer, XII, 10, p.595.
96 Theon of Smyrna, *Exposition,* translation by *5.* Dupuîs, p.3PE; Theon of Smyrna, *Exposition,* translated by J. Delattre, p.185.
97 In 0º. Ptolemy, *Almagest,* translated by G. J. Toonier, XII, lO, p.595.
98 In @ G“. PioJémée, */’Alzoagestr.,* translation by G. J. Toomer, XJI, 10, p.596.

IV - Rising and setting of stars and pplanets

Theon and Ptolemy also discuss a subject that partly relates to synodic phenomena: the rising and setting of celestial bodies. This is another "classic" problem, which had been of great importance to Babylonian astronomers in particular.

As Theon points out, several types of rising and setting must be distinguished. [9] First, all celestial bodies—fixed stars, planets, and luminaries—rise and set as they pass above or below the horizon. [1°] This corresponds to what is generally meant by "rising" and "setting" in reference to the sun.

Lever Setting



Horî zon East Western horizon

Fig.

Although Theon does not say so explicitly, it goes without saying that for him this phenomenon is due to the diurnal motion caused by the rotation of the sphere of fixed stars. A celestial body thus rises in the east and sets in the west every day, provided that it is far enough from the sun to be visible at the times in question.

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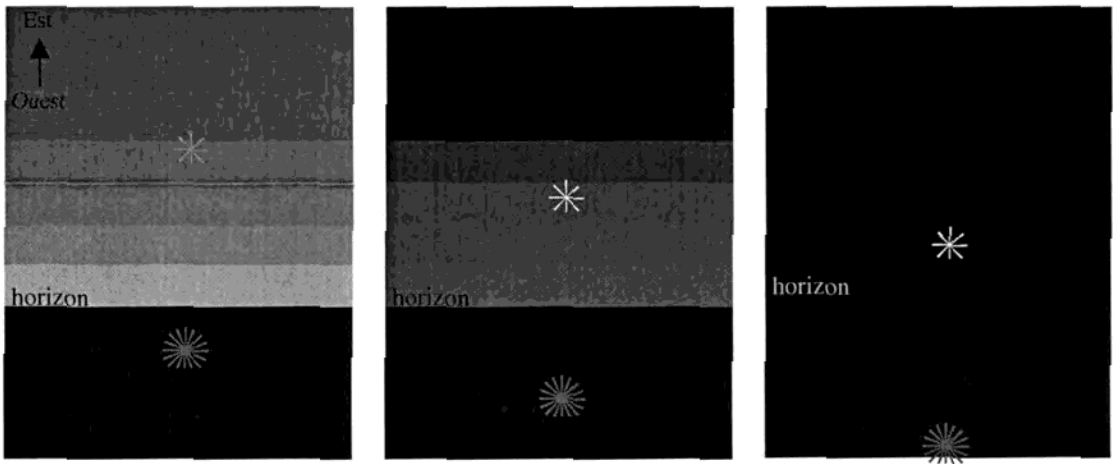
99 Theon of Smyrna, *Mposilion,* translated by J. Dupuis, pp. 225-227; Theon of Smyrna, *Mposiiion,* translated by J. Delattre, pp. 130-131.
100 Theon of Smyrna, *Exposition,* translated *by I.* Dupuis, p. 225; Theon *of* Smyrna, *Emp* siiiDZt*,* translated by J. Delaitre, p.130.

Secondly, a star already in the visible part of the celestial sphere can rise in the following sense: it **was** invisible because of the brightness of the sun, and it becomes visible because the luminary sets. It is in this sense that stars and planets that become visible at sunset 'rise'.

Rising of a celestial body in the evening (western horizon):



The celestial body is not yet visible | Rising of the celestial body | The celestial body is clearly visible

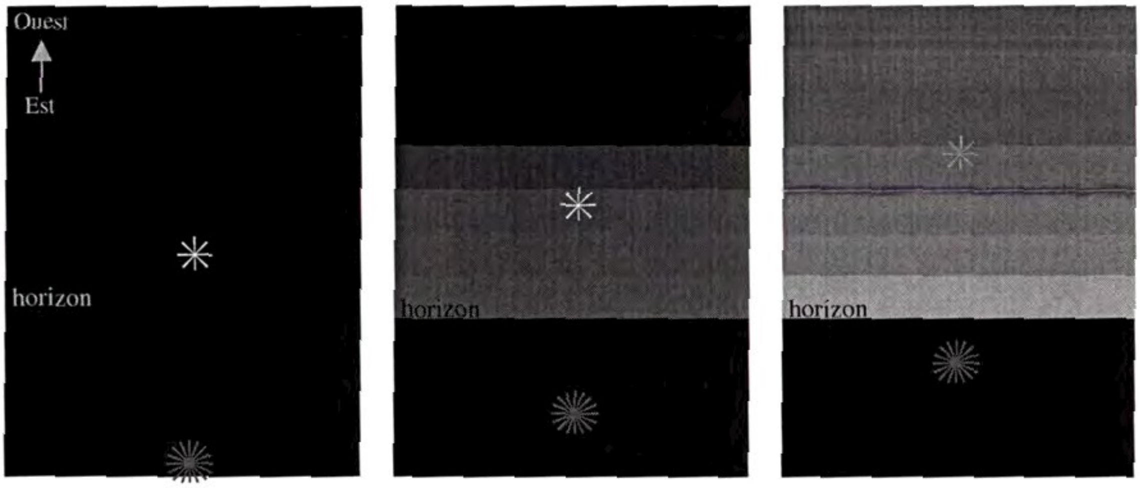
Fig.4.16

Similarly, a celestial body that is visible because it is still dark, or because the dawn sky is not yet bright enough to drown out its light, disappears when the brightness intensifies with the rising of the sun. Theon also considers this case—which of course applies to all stars and planets above the horizon when dawn arrives—as a "setting."(02)

IO l Theon of Smyrna, *Mpo.‹ition,* translation by J. Dupuis, pp.225-227; Th6on dc Smyrne, *Exposi'ti'on,* translation by J. Dclattre, p.131.

~~102 Theon of Smyrna, *Exposition,* tra~~nslation by J. Dupuis, p.225; Theon of Smyrna, *ExpOSftion,* translation by J. Dclattre, p.1 S 1.

Setting of a star in the morning (eastern horizon):



The star is still clearly visible visible

Sunset of the other

The star is no longer visible

Fig.4.17

Theon distinguishes a third category of rising and setting of celestial bodies, [103] but this is in fact a special case of the two situations we have just seen. We could even say that this third type of rising and setting occurs when the first two occur at the same time. This is indeed possible for celestial bodies in opposition to the sun. When the sun sets, the celestial body that is diametrically opposite it passes above the eastern horizon. As the sun sets,

103 Theon of Smyrna, *Mposiiion,* translation *by 1.* Dupuis, p.225; Theon of Smyrna, *Exposition,* translation by J. Delattre, pp.10-131.
~~104 Theon refers to the star "diatnem~~ially opposite" to the sun. He therefore does not take into account all the *stars rising* above the horizon to the south and north of this point. Theon of Smyrna, *Exposition,* translated by J. Dupuis, p. 225: Theon of Smyrna, *Exposition,* translated by J. Delattrc, p. 130.

if it is sufficiently bright, the star becomes visible. Similarly, at dawn, when the sun rises, the star that is opposite it disappears behind the horizon.

West. Theon seems to consider that only one celestial body at most can be in this position at any given dawn or dusk. We can therefore assume that he reserves this definition of sunrise and sunset for celestial bodies that are *exactly* opposite the sun.

As with sunrises and sunsets on the horizon, those caused by differences in

The brightness of the sky due to sunrise or sunset is also likely to recur at intervals of one day for the same celestial body. However, Theon also refers to sunrises and sunsets in a different sense than diurnal: these are known as heliacal risings and settings."[7]These are due to the movement of the sun on the ecliptic and, when it is not a fixed star, to the movement of the celestial body itself. They are special cases of the first two categories of risings and settings that we have seen.

The phenomenon is both simpler and relatively similar for fixed stars and superior planets. These celestial bodies always seem to be "caught up" and "overtaken" by the Sun as it moves from west to east: the former because they are fixed and the Sun's movement is defined in relation to them, and the latter because, although their movement along the ecliptic is generally from west to east, it is slower than that of the Sun. Thus, in their case, there is a heliacal setting and rising approximately once a year. A heliacal setting occurs before they enter into conjunction with the sun, when the latter has moved close enough to them in the west that their elongation no longer allows them to be visible. For stars and superior planets, a heliacal setting therefore always occurs when the celestial body is east of the Sun. It can then only be seen in the evening, because in the morning it does not pass above the horizon until after the Sun. Consequently.

105 Since the eastern horizon is not very dark at sunset, many stars that are visible to the naked eye in a clearer sky are not yet visible. The three planets in question—Mars, Jupiter, and Saturn—will of course be visible.

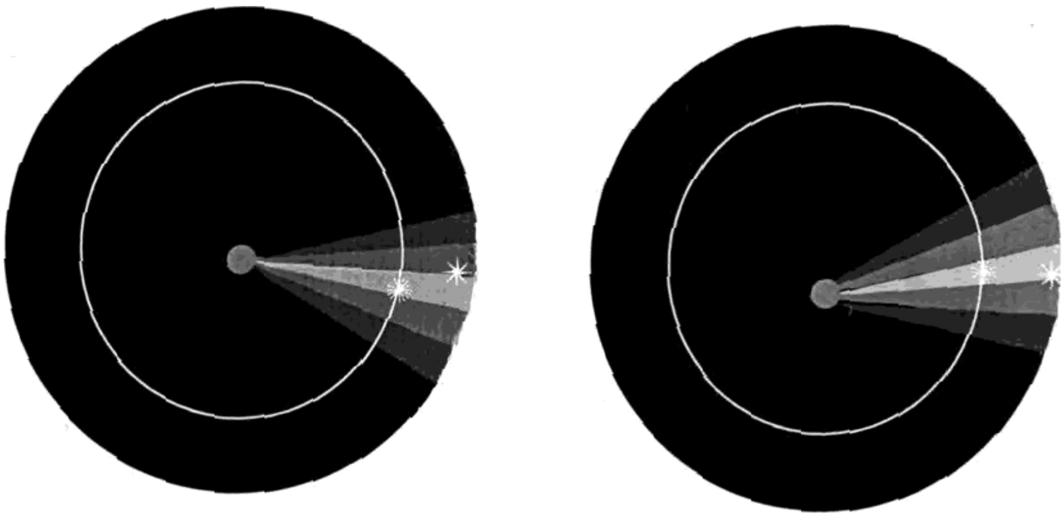
106 This, together with the fact that the third category of loss is in fact only a special case of the other two, would suggest that the star in question was considered to be of particular importance.

107 Theon of Smyrna, *Mposiiion,* translation by *I.* Dupuis, pp. 225-227; Theon of Smyrna, *Exposition,* translation by J. Delattre, pp.130-131.

its last appearance i.e. its heliacal setting — will consist of a setting behind the horizon. shortly after that of the sun." Similarly, for fixed stars and superior planets , a heliacal rising occurs after a conjunction with the sun, when the latter has moved far enough eastward for the elongation of the celestial body

allow it to become visible again. For these celestial bodies, a heliacal rising *therefore* always occurs when they are west of the sun. In this case, they can only be seen in the morning, since in the evening they pass behind the horizon before the sun. ([1]) Consequently, their first appearance will consist of a rising above the horizon, shortly before the sun rises. Theon uses the example of Sitius to illustrate this case. ([1]) '([0])

Fixed stars



setting rise

Fig.4.18

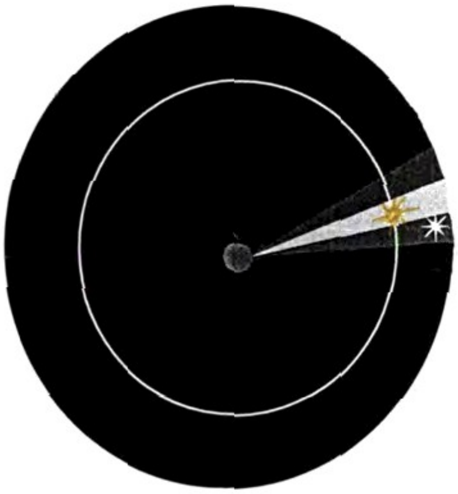
108 Theon of Smyrna. *Exposition,* translation by J. Dupuis, p.227; Theon of Smyrna. Exposition, translation by *1*. Delaitre, p.131.

109 Theon **of Smyrna,** *Mfi›osilion,* translated by J. Dupuis, y.22S; Th5on of Smyrnc, WjPuxfrza/z, translated by /. Dclattre, y. ISO.

\ IO Th6on of Smyrna, *£fiqmsiiion,* translation by J. Dupuis, y.227; Theon of **Smyrna,** *Exposition,* **translated** by J. Dclattre, y. I? I .

Stipcrieiires planets

setting

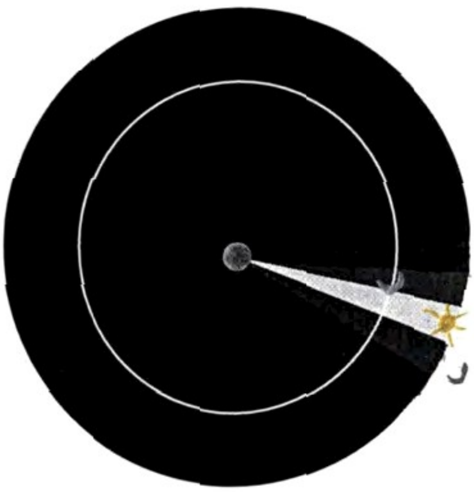


rise

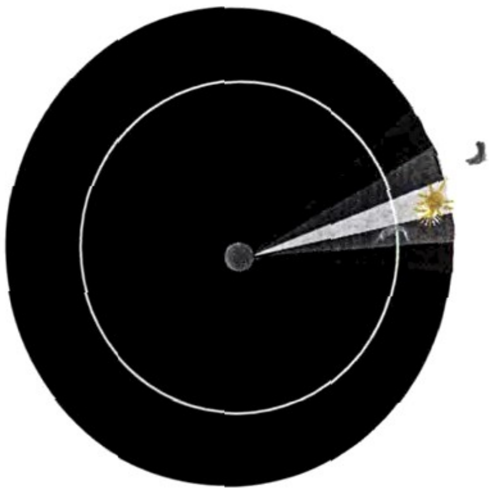
Fig.

La The moon presents the opposite case, because its movement from west to east on the ecliptic is faster than that of the sun. It therefore catches up with and overtakes it to the west once a month.

moon



C OtlCîl CI'



lever

Fig.

As with the previous celestial bodies, its heliacal setting always occurs before it enters into conjunction with the sun—i.e., just before the new moon. But at this point it is in the west, not in the east of the daytime luminary. It is therefore visible in the morning. ([1)] ' This is why, before the new moon, the moon is a morning star.[112] Its heliacal setting takes place as it disappears in the brightness of the rising sun. [(13)] Its heliacal rising occurs when, having passed the sun on the ecliptic, it is east of it and therefore visible in the evening. [(14)] Thus, after the new moon, it is an evening star.[5] It will therefore make its first reappearance in the growing darkness of sunset. [6] It should nevertheless be noted that for its heliacal sets and rises, the moon, although it appears and disappears due to the varying brightness of the sky, is nevertheless just above the horizon. In the case of a heliacal setting, it has just risen above the horizon; in the case of a heliacal rising, it is about to disappear behind it. Theon does not specify this aspect of the phenomenon, unlike Ptolemy, who uses it implicitly in his demonstrations concerning the inferior planets, which are in a similar situation during their movement from west to east—as we shall see. The reason for this is illustrated by the following figures:

111 Theon of Smyrna, *Exposition,* translation by *1.* Dupuis, pp. 223-225; Theon of Smyrna, *Exposition,* translation by J. Delattre, pp.129-130.

112 Theon of Smyrna, *Exposition,* translation by *1.* Dupuis, p.225; Theon of Smyrna, *Exposition,* translation by J. Delattre, p.129.

113 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.227; Theon of Smyrna, *£rposilion,* translated by J. Delattre, p.131.

1 14 Theon of Smyrna, *Exposition,* translation by J. Dupuis, pp. 223-225; Theon of Smyrna, *Exposition,* translated by J. Delattrc, pp. 129-130.

115 Theon of Smyrna, Erposifioo, translated by J. Dupuis, p. 225; Thëon of Smyme, *Exposition,* translation by J. Delattre, p.1 29.

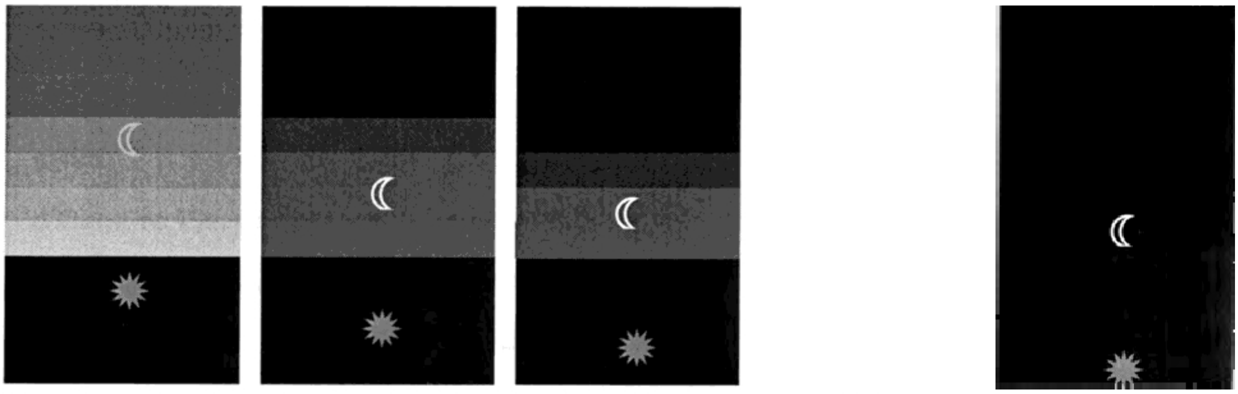
11s Theon of Smyrna, *Exposition,* translation by J. Dupuîs, p.227.; Theon of Smyrna, *Exposition,* translated by J. Delattre, p.131.

lever : soir horizon vers l'Ouest
après nouvelle lune

Day considered:

Rising of the moon

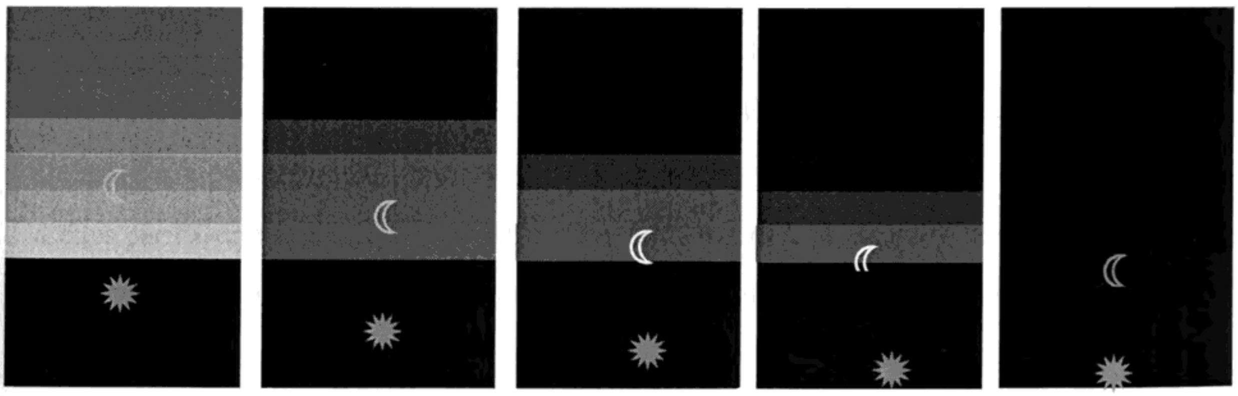
The day before:



Rising of the Moon

lever héliaque

Ï.*two days earlier:



Rising of la moo n Heliacal rising

Direction of time diume
(i.e. in the same evening)

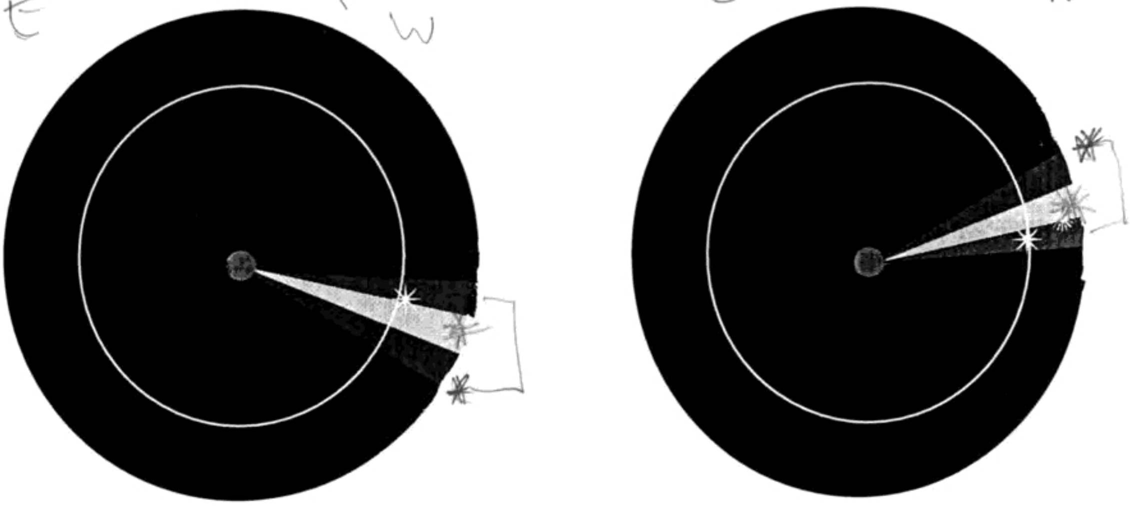
We can see that a star can of course appear when it is already well above the horizon on a given evening, but this means that on the previous evening, although *its* elongation is less, it still appears; it simply does so at a slightly later hour, when the sky at its height has become sufficiently dark. Its heliacal rising therefore occurs on the evening when it becomes visible in the darkness just before passing behind the horizon: the first evening when its elongation is sufficient for it to be above the horizon at the moment when the latter is just dark enough for the celestial body to be visible. The opposite is true for heliacal setting.

The case of the inferior planets is more complex because their elongation is limited, meaning that they are always close to the sun and appear to move back and forth between the east and west of the luminary. As a result, their heliacal risings and settings can occur either in the evening or in the morning. There are four possible scenarios. When they *move* from west to east, these planets behave like the moon, because this movement is faster than that of the sun. They then experience a heliacal setting in the west of the sun in the morning, and their last appearance ends with the increasing brightness caused by the rising of the luminary. Similarly, their heliacal rising occurs when they are east of the sun, making them evening stars, and occurs in the increasing darkness that accompanies *the setting* of the sun. On the other hand, when Venus and Mercury are in retrograde motion, from the east and west, they are in a similar situation to the superior planets and the fixed stars, because they see the sun approaching them from the west *and* moving away from *them* in the east. They are located east of the luminary at their heliacal setting, which therefore takes place in the evening, and they disappear behind the horizon after the sun. Their heliacal rising takes place when they are west of the sun, in the morning: they reappear above the eastern horizon before sunrise.

1 l7 Theon of Smyrna, *Exposition,* translation by *1.* Dupuis, p.225. Theon of Smyrna, *Exposition,* translated by J. Delattre, p.130.

Inferior planets
iiiouveinent d'Otiest en b.st

E W E W

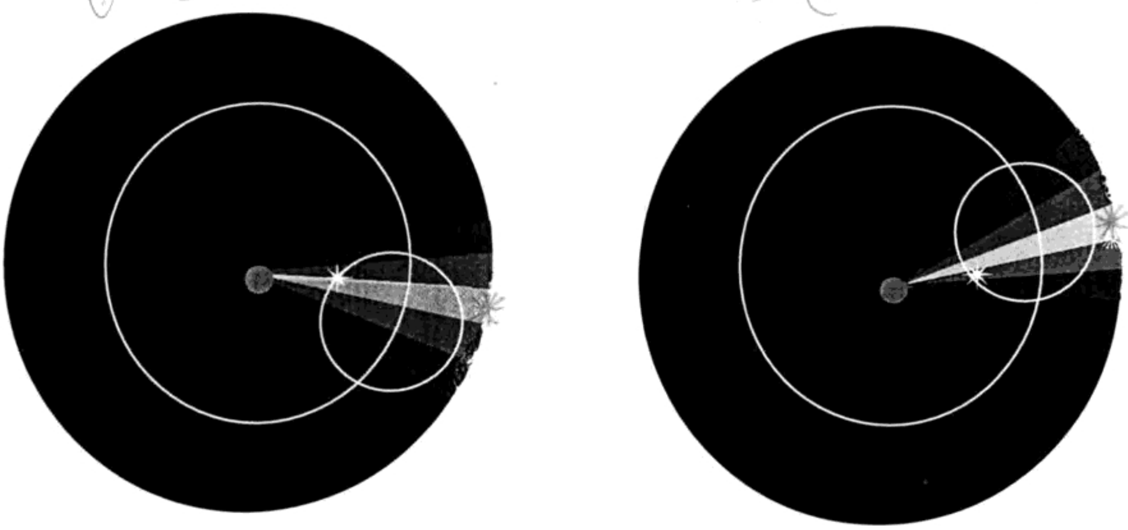


setting lever

Fig.4.22 i.e. like moon

Inferior planets
inoiii'eineiit réiroçn'ade

E W W



lying lever

big.4.23 i.e. like stars & outer planets

Although Theon does not explain the heliacal risings and settings in as much detail as we have done here for the sake of clarity, it is perfectly clear that he understands them in this way.

Ptolemy agrees with Theon on all these points, which is hardly surprising. He deals more specifically with heliacal risings and settings. However, unlike Theon, he does not address the problem for fixed stars and planets at the same time. He does so at the end of Book VIII, which is devoted to these stars—as is Book VII—while for the planets, he first needs to develop his theory of latitudes. He therefore only addresses this question at the very end of the Almagest, in Book XIII.

What Ptolemy seeks to determine are the values of the elongations of celestial bodies during their heliacal risings and settings. Indeed, the first and last visibilities of a given celestial body occur at the same angular distance from the Sun below the horizon, measured along the vertical, *I''arcus visionis"* (in yellow in Fig. 4.25 below). This distance varies from one star to another due to their differences in brightness. Ptolemy determines its value for each of the five planets, but does not do so for any fixed stars, not even the most important ones. Furthermore, for the same celestial body, this distance does not correspond to a single elongation: the elongation varies according to the inclination of the ecliptic; for planets, it also varies according to their movement in latitude."

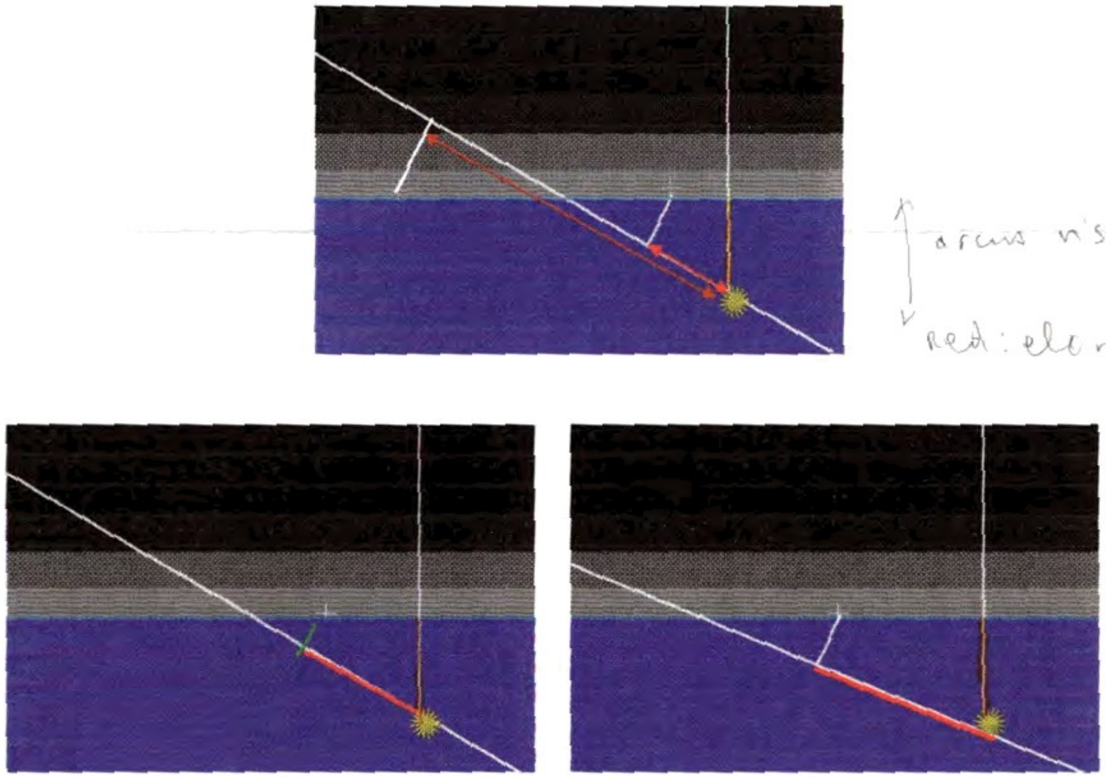
11g Ptolemy, *Almagest,* translated by G. J. Toomer, XIII, 7-10, pp.636-647.

119 Ptolemy, *Almagest,* translation by G.J. Toomer, XIII.7, pp.636-637. The fact that 1* *arcus visioni,i* is always the same for the same star is due to the fact that for a given "depth" of the sun below the horizon, the brightness along the latter varies little for the limited distances that can separate it from the vertical of the sun, precisely because Venus and Mercury themselves have limited elongations.

120 Ptolemy, *Almagest,* translation by G.J. Toomer, XOI.7, pp.637-638.; Ptolemy considers observations made when the point on the ecliptic that intersects the horizon is G 0 , because in this case the angle between the ecliptic and the horizon is the same at dawn and dusk.

121 The reason for this is that he simply does not see the point in doing such important work—phenomenal if he had to do it for all the fixed stars visible to the naked eye. The precession of the equinoxes would eventually render it obsolete, and in any case Ptolemy doubts the importance of these elongations as a meteorological factor. Ptolemy, *Almagest,* VIII.6, translation by G.J. Toomer, pp.416-417.

122 Ptolemy, *Almagest,* XIII.7, translation by G.J. Toomer, p.637,



. 4.25

The inclination of the ecliptic varies with geographical latitude and with the longitude of the point of intersection between the ecliptic and the horizon. [123]However, when tabulating the elongations of the planets, Ptolemy does not express these elongations in terms of the latter longitude and the latitude of the celestial body, but in terms of the positions of the planets. [124] He constructs, for a given geographical latitude, [124] tables indicating the elongations of each planet at its heliacal rises and sets [125] for twelve positions of these planets on the ecliptic—at the beginning of each

123 Ptolémée, Almageste, XIII.7, translation by G. Toomer p. 637.
124 of inclination of the ecliptic being, let us recall, function of the latter. Ptolémée, Almageste XIII.7. translation by G J. Toomer, p *8
125 Respectively in the evening and in the morning for the superior planets For Mercury and Venus. Ptolémée considers the heliacal settings in the morning and in the evening. The same applies to the levers héliaques

sign.'[26] 11 does not explain how he calculated these values, despite the fact that, as Pedersen notes, they seem to be mainly illustrative.

In addition to providing these tables, Ptolemy reports two phenomena concerning the inferior planets. Although Theon makes no mention of them, they were clearly well known to astronomers. The first concerns the phases of Venus, which, according to Proclus, appeared quite mysterious.[2]'The intervals between the heliacal settings and risings of this planet vary greatly: they can range from two days—when the planet is near the beginning of Pisces—to 16 days—around the beginning of Virgo.'[2] 'Ptolemy accounts for this difference on the basis of the value he obtained for the arrus *visionis.* The second phenomenon concerns the behavior of Mercury: the latter can have a heliacal rising and setting on the same side of the sun. Ptolemy explains this by the fact that the planet experiences a heliacal rising followed by a heliacal setting on the other side of the sun, but that these are not visible. This explanation is clearly not original to him, as he refers to Mercury's "missing phases"without further explanation. His contribution consists in demonstrating that Mercury's elongation, when it is in certain signs towards the beginning of Scorpio and Taurus"[0] —, is less than that which the planet should have in order to be visible."'

As might be expected, Ptolemy and Theon's conceptions of the rising and setting of the stars are therefore similar in broad outline. Theon simply places greater emphasis on one type of rising and setting, that of stars in opposition to the sun—i.e., the third type we have described—for which Ptolemy shows no particular interest. Ptolemy provides an explanation of the peculiarities of the phases of Mercury and Venus, which Theon does not mention.

126 Ptolemy, *Almagest,* XlR.10, translation by G.J. Toomer, pp.646-647.
127 0. Neugerbauer, *A History of Ancient Mathematical Astronomy, vol. I,* p. 239.
128 Ptolemy, *Almagest,* XIII.8, translation by G.J. Toomer, pp.640-644. This is in fact the point of intersection between the ecliptic and the horizon found at these longitudes in Ptolemy's demonstrations, and not the plane itself.
129 i.e. first and last visibilities.
130 As with Venus, these longitudes represent those of the point of intersection between the ecliptic and

131 Ptolemy, *Almagest* XIII.8, translation by G.J. Toomer, pp.644-645.

No mention. But Ptolemy's contribution once again consists in the elaboration of his tables.

Apart from the elongations of Mercury and Venus, which cannot be represented in the same way in Theon's preferred hypothesis as in Ptolemy's, the two astronomers share the same general views on the phenomena we have described in this chapter. Ptolemy's contribution in this regard does not consist in an original way of accounting for them, *but* in the precision with which he can do so and predict them. The tables he developed for this purpose are based on the geometric models he had previously developed, and this is where his originality lies. Better quantitative accounting for these various phenomena is an application of his models and a way of demonstrating their relevance, if not their **validity.**

CHAPTER V: PHYSICAL ASTRONOMY

After the publication in 1908 of Pierre Duhem's book, ri r re ‹irai roysvo, it was long believed that in ancient times, mathematical astronomy and physical astronomy were two distinct, even rival schools of thought. This was indeed Duhem's thesis, for whom Ptolemy was a representative of the former, while Theon represented the latter: as we shall see, Theon presents physical models after his mathematical models, to which he attaches great importance. Crucially, for Duhem, the difference between their approaches did not simply consist *of* a difference in methods for understanding the same phenomena, but was intimately linked to opposing epistemological positions. He thus contrasted Theon's "realism" with Ptolemy's "positivism," to use modern terms. Duhem bases this on a series of excerpts from *Book XHI* of */'A/sagciie,'* which we will offer a different reading of at the end of this chapter. But it seems clear that he also bases his argument on the fact that */'A/iuagesfe contains* very few references to physical models. A t the time Duhem wrote K *Si r ra QOf vD,ZfEva,* only the beginning of Ptolemy's *Book of Hypotheses*, transmitted in Greek, was known: the first part of the first treatise, which mainly reproduces, in condensed form and with a few modifications, the results obtained in *the Almagest* — models and parameters. Goldstein's discovery in 1967 of a complete version of *the Book of Hypotheses* transmitted in Arabic suggests a completely different

1 Pierre Duhem, Jryrri ro *‹yai vote va,* pp.13-20.
2 Pierre Duhem, her r re pifi/ c›yrrn, pp. l 7- i 9.
3 On a suggestion made by Hartner in 1964.
4 as far as we can judge. *It* consists *of* two treatises.

appreciation of Ptolemy's work and epistemological positions than that put forward by Duhem. Indeed, the second part of the first treatise is based on a physical argument for calculating the distances of the planets, and above all, the second treatise is entirely devoted to physical questions, setting out the arrangement of the spheres corresponding—with a few modifications—to the mathematical models developed in *the Almagest.* Clearly, Ptolemy was not uninterested in physical astronomy. For his part, Theon, after presenting the mathematical models we have described, proposes physical models—the importance of which he effectively emphasizes, as Duhem says. As we shall see, this does not imply that the question of the relationship between mathematical astronomy and physical astronomy in Hellenic antiquity became meaningless. Nevertheless, before addressing this subject in more detail, we will begin by briefly presenting the physical models of Theon and Ptolemy.

eIuh dè u s

Since Theon's mathematical models are almost identical for all planets, as might be expected, his physical models are also similar, apart from a very particular arrangement that Theon proposes for the spheres of Venus, Mercury, and the sun.

It should be noted that Theon proposed *only* one physical model. While mathematically he represented planetary motions indifferently by the epicycle hypothesis and the eccentric hypothesis, physically he considered only the materialization of the epicyclic hypothesis. It seems clear that he could not conceive of a similar materialization for the eccentric hypothesis. We will attempt to determine the reasons for this below.

Theon's physical model is therefore as follows:

5 Theon of Smyrna, Mypsiriori, translation by J. Dupuis, p.293

Fig. 5.1

Theon considers two types of "spheres": "hollow" [xoiXq] and "solid" [crrcpsn]. His use of these terms suggests that he does not mean "surface" when he says "sphere," as in epcrr, for example. For him, a solid sphere is a sphere considered in its entire volume—i.e., from its center to its surface—and a hollow sphere is a spherical shell considered in its thickness.

Thus, in this diagram, the hollow sphere n§y6-epm represents the sphere of the fixed stars, and rotates from east to west around the axis of the equator. Theon does not specify whether he conceives of the stars as being distributed on one of the faces—inner or outer—or inside the thickness of this sphere. Nor does he speculate on the value of this thickness in relation to that of other spheres, for example. He obviously has no observational astronomical arguments to take a position on these questions, but he does not put forward any physical arguments either.

6 Theon of Sinytne, *Exposition,* translation by *1.* Dupuis, p.292, 1.17.
7 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.294, t.l.
8 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p. 293.
9 Theon of Smyrna, *Exposition,* translation by *1.* Dupuis, pp. 293-295

Inside this sphere o9yô-cpoi is another hollow sphere, sp‹rr-

•x•y, which is concentric. This sphere acts as an inclined deferent: it revolves around the axis of the inclined celestial sphere and is responsible for movement in latitude. Theon does not explicitly take sides on either of the two possibilities regarding its movement: it can rotate from east to west like the sphere of the fixed stars, less quickly than the latter, so that a given point on opor-n2uy appears to move from west to east relative to the fixed stars, i.e., it is "left behind"; or it may also be animated by a movement from west to east·This last possibility implies that nbyõ-spox would then be solid with the sphere of fixed stars, while having its own movement relative to it.

Within the thickness of epa-n2uç lies the solid sphere eÇnp, which materializes the epicycle and rotates on its own axis. In general, the stars are located on the surface of such solid spheres, as rei at point e.

The fact that the sphere n§y6-epnz represents the sphere of the fixed stars suggests that the other two spheres correspond to the model of Saturn, but Theon does not specify this. He clearly intends to represent a general model, which raises the following question: should we assume that for each planet, there is a sphere corresponding to o§yô-cpoi outside the sphere corresponding to spot-nbsp, just inside the spheres specific to the planet above it? The fact that Theon gives no details about the position of the stars on or in n§y6-Epoi would suggest that this sphere does not represent the "sphere of the fixed stars" itself, but rather a type of sphere that would be found for each planet and would materialize the diurnal motion coupled with that of the sphere of the fixed stars. Such a hypothesis is not as extravagant as it seems because, as we shall see, this is exactly what Ptolemy envisages. Nevertheless, this does not seem to be the case with Theon, who says elsewhere:

10 Theon of Smyrna, *Expo.Kihon,* translation by J. Dupuis, p.295.
11 Theon *of* Smyrna, *Exposition,* translation by J. Dupuis, pp.293-295.

"We must believe that for each planet there is a hollow sphere of its own, which carries within its thickness a solid sphere, and that the solid sphere in turn carries the other on its surface."

He makes no mention of a third sphere outside the hollow sphere in question. Furthermore, referring to the same type of model for Venus, Mercury, and the sun, he speaks of only "two spheres" for "each of these stars," one hollow and one solid·

Theon further specifies the configurations of his model for which the celestial body is at its maximum, minimum, and average distances when the speed of rotation of the solid sphere cÇzrJ is equal to that of the sphere ppm-n/uç, i.e., in the case of the sun. These distances correspond respectively to the following situations: solid sphere at p and celestial body at e, solid sphere at v and celestial body at u, solid sphere at ‹p or c› and celestial body in L or Ç [4]

For celestial bodies other than the sun, the rotational speeds of the solid sphere eÇttrj and the hollow sphere epos-ayc›y differ, in accordance with what was said in chapter 3 about the speeds of the planet on the epicycle and of the latter on the deferent.

If we compare Theon's 'standard' model, which we have just described, with that found in Ptolemy, we find a number of differences, which are certainly not unrelated to each other. Indeed, they are more or less directly attributable to the fact that, unlike Theon, who sees the eccentric hypothesis as a simple optical effect with no physical reality, Ptolemy cannot do without a physical explanation for this hypothesis, since it occurs in almost all his models at the same time as, and not as an alternative to, the epicyclic hypothesis. This necessity is most certainly what led him to develop the following model:

12 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.301.
13 Theon of Smyrna, *Mposiiion,* translated by J. Dupuis, p. 301 [XXXf II].
14 Theon of Smyrna, *Wposilion,* translated by J. Dupuis, p. 297.

Fig.5.2.a

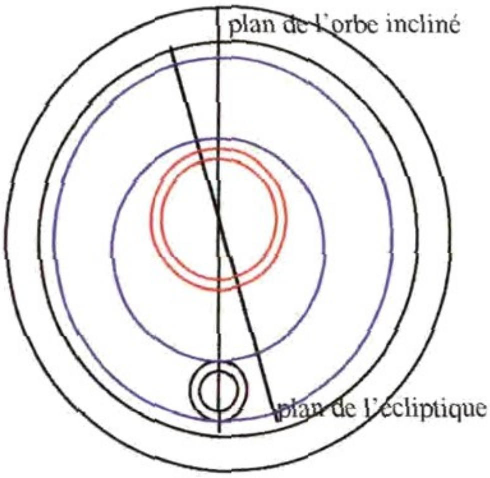


Fig.5.2.b

We find a sphere that represents the epicycle—although it differs from Theon's, as we shall see—inside a hollow sphere which, as in Theon's model, acts as an inclined deferent, although in Ptolemy's model this inclination only partially accounts for variations in latitude. The essential difference between this model and that of Theon is that for each star, this hollow sphere is surrounded by two other 'spheres.' It is the outer surfaces of these last two spheres that are concentric, while the hollow sphere has as its center that of the eccentric deferent. The movement of the sphere surrounding the eccentric sphere is coupled to that of the sphere of the fixed stars by means of a sphere surrounding it. The two 'solid spheres' clearly serve to facilitate the 'nesting' of the models of the different celestial bodies within each other, and their difference in eccentricity makes it very tempting to 'recenter' the model at each level in this way.

Pto emee Régis Morclon (notes). pp. 14-19

16 As can be seen in Fig. 5.2, these spheres have the distinctive feature of varying thickness: this lack of ~~symmetry is the reason for our quo~~tation marks.

15 l' ', *le Livre des Hypothèses*, Traité 2, traduction de

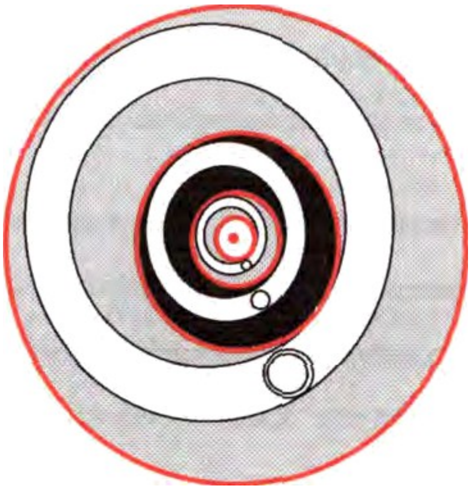


Fig. 5.3

The need to develop such a model means that Rolémée is not forced to represent the movement of the sun using the epicyclic hypothesis. Thus, for the model of the sun, he does not use an equivalent of the solid sphere emmy, but only a hollow, off-center sphere ‘(7)

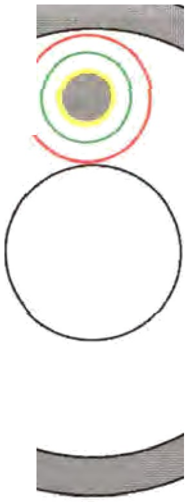
This possibility is difficult for Théon to accept. Indeed, once he can use the model we saw above—with concentric deferents, of course—postulating an eccentric hollow sphere is unnecessarily complicated, as this would require him to add two "spheres" rather than one—i.e., the solid sphere corresponding to the epicycle. Furthermore, the two "spheres" in question have the disadvantage of lacking symmetry. This would probably bother Theon, who conceives of spheres not as surfaces but in their thickness: it is doubtful that he would actually consider bodies defined by two spherical surfaces but nevertheless endowed with a variable thickness, which Rolémée is forced to use, to be "spheres." These are undoubtedly the reasons why Theon considers that there is only one possible physical model, corresponding to the epicycle hypothesis. For Theon, this resolves the uncertainty in which

J 7 Piolémée, *The Book of Hypotheses.* Traile 2. Translation by Régis Morelon (notes). p. l4.

The equivalence of the two mathematical hypotheses led the astronomer to conclude: "... it [the movement] follows the epicycle, even though it appears to follow the eccentric."

On the other hand, perhaps the most interesting aspect of what Theon proposes as physical models is one of the alternatives he offers concerning the arrangement of the spheres of the Sun, Venus, and Mercury. The first of these alternatives corresponds to the "standard" model we have described, which he applies to each of these celestial bodies individually. He then accounts for the limited elongation of Venus and Mercury by postulating that their solid spheres are animated by the same speed of revolution due to the movement of their respective hollow spheres. This is undoubtedly the most obvious way of giving the two planets average speeds equal to that of the sun when developing models for each celestial body individually: the same model is used for all celestial bodies, and the parameters are adjusted for each one. However, Theon proposes an alternative physical model that allows for the revolution of Mercury and Venus around the sun (a movement that we discussed in Chapter 3). Thus, Theon imagines that there may be only one hollow sphere for the three bodies, containing three spheres (0) nested inside each other, each carrying a body: the sun is thus fixed to the middle one, and the order of the other two planets is clearly determined by their elongations: "the maximum elongation being smaller for Mercury, this planet is located on the middle sphere, and Venus on the outer sphere. [2]

lfi Theon of Smyrna, *Mposifion,* translation by F. Dupuis, p. 299, lines 29-30.
19 See also ch. 3, p. 83. 11 Considering this model, it is difficult not to think of another astronomer who, much later, would postulate an arrangement not unlike this one, namely Tycho Brahe, who this time would place all the planets in revolution around the sun.
20 These three spheres naturally represent the epicycles, but they can no longer be described as solids, since the two outermost spheres must obviously be hollow.
21 Although Theon does not explicitly refer to this in the positions he assigns to Mercury and Venus.
22 Theon of Smyrna, *Exposition,* translation by J. Dupuis, pp. 301-303.



sphere of the sun
sphere of Mercury
sphere of Venus

Fig. 5.4

This configuration has the advantage of providing a simple and elegant way of representing maximum elongations. The fact that the three celestial bodies have the same average speeds appears to be a natural necessity rather than the effect of arbitrary parameters. Of course, it introduces a certain diversity into the models of the different celestial bodies, which could be perceived as a lack of harmony. Nevertheless, the difference in the behavior of the planets with regard to elongation is an observational fact that suggests their classification into two types: Ptolemy himself mentions this motivation for separating the planets into inferior and superior. Furthermore, this diversity is not as significant as that introduced by Ptolemy and his various models; the standard model remains the same in Theon, although the three celestial bodies share the same hollow sphere (corresponding to the deferent) and their solid spheres (corresponding to the epicycles) are nested within each other.

However, as we said in Chapter 3, "the two configurations that Theon proposes for the sun, Venus, and Mercury are not equivalent from the point of view of

_J see 3 pppö-pfv

: they do not constitute two ways of accounting for the same phenomena, between which one would choose according to mainly aesthetic criteria. If Theon allows himself to propose this second model and even, it seems, to give it his preference, while Ptolemy does not even allude to it, the reason seems to lie in a disagreement between them on the observations: the second model proposed by Theon is based on the notion that Mercury and Venus eclipse each other, whereas the first is not compatible with this but requires, on the contrary, that it is always the same planet that eclipses the other.[5] As we have seen, Ptolemy believes that Mercury always eclipses Venus and not the other way around,[2] 'although other arguments also lead him to favor the configuration he chose for the order of the planets, and it is difficult to estimate which arguments he finds more convincing. Nevertheless, the fact that Ptolemy does not even consider the possibility of the second model proposed by Theon would suggest that he was not aware of it and had not thought about it. This may be attributable to the attention he pays to the mathematical models of each of the stars, which undoubtedly makes it more difficult for him to conceive of possible relationships between their models.

The "standard" physical models of Theon and Ptolemy present yet another difference, which reflects [the]mathematical effort made by Ptolemy to account for variations in the latitude of the planets. However, the physical model proposed by Ptolemy in *the Book of Hypotheses* is not the materialization of the theory of latitudes that he developed in *the Almagest,* but rather a greatly simplified version of it. The spheres that correspond to the inclined circles in *the Book of Hypotheses* always retain the same inclination: '([7]) there i s no longer any variation in the angle between the deferent and the ecliptic, as was the case for Venus and

24 Theon of Smyrna, *Exposition,* translation by F. Dupuis, p.303, 1.4-6.
25 The two models also imply, of course, another difference: according to the second, the sun eclipses Venus and Mercury as much as the opposite; but, as Ptolemy points out, it is illusory to claim to distinguish the two phenomena observationally; Theon makes no reference to this, but this fact alone would suggest that he shared Ptolemy's opinion on this point.
26 See ch. 3, pp. 85-86.
27 Pedersen, p.392. This inclination is 0; 10° for both planets.

Mercury." The movements of the sphere that materializes the epicycle now constitute the entire difference between Ptolemy's theory of latitudes and that of Theon. These movements are also simplified: for the inferior planets, the angle of this sphere to that of the deferent is constant; it is not constant for the superior planets, but for these its maximum value is still equal to that of the inclination of the deferent plane to the ecliptic. To account for these movements, Ptolemy's physical interpretation of the epicycle is naturally a little more complex than Theon's. While the latter only needs a solid sphere, Ptolemy must postulate two spheres, one inside the other. Curiously, Ptolemy fixes the star not on the surface of the inner sphere but slightly inside it.

II I **ication** s ue d st n e **des** **rbes**

Ptolemy's physical conception of the heavens allowed him to calculate the distances between celestial bodies. Although Theon does not mention this possibility, it is also consistent with his physical models. Indeed, the assumption on which Ptolemy based his calculations was that the spheres of the different celestial bodies were contiguous. Ptolemy's argument in favor of this thesis is physical: he believes that a vacuum cannot exist. This notion may seem reminiscent of that of the Eleatics, but in fact Ptolemy's argument is different: he does not refer to a logical impossibility in conceiving that a vacuum, i.e., "nothing," can exist. For Ptolemy, the argument corresponds more to Aristotelian views: nature does nothing in vain, so it is unlikely that there is a space that serves no purpose·As Pedersen notes, the fact that this argument is relevant to Ptolemy's

2s see ch. 3, p. 122.

29 Ptolemy, *The Book of Hypotheses,* translated from Arabic by R. Morelon, in *MIDEO,* 21 (1993), p.72

simple probability is undoubtedly one of the reasons why he deals with the problem of planetary distances in *the Book of Hypotheses* rather than in *the Almagest:* "He seems to group together problems to which he attaches less certainty because they are based more on physical hypotheses than on geometric models developed as directly as possible from observations."[3]

Theon does not address this subject, but the fact that he assigns only two spheres to each star would suggest that he does not envisage any space between the spheres of different stars. Furthermore, when he describes the arrangement of three interlocking spheres for the sun, Venus, and Mercury, he specifies that the sphere of Venus "would fill the entire thickness of the common hollow sphere." This seems to indicate that his views on the possibility of useless spaces are similar to those of Ptolemy.

To calculate the distances of the planets, Ptolemy based his calculations on the values he obtained obtained in *the Almagest* for the distances of the moon and the sun, and on his ratios of the maximum and minimum distances of the other stars, which he can easily obtain from his geometric models. For those parts that have already been mentioned, he *thinks* that beyond the moon is Mercury, and that Venus comes next, before the sun. His hypothesis on the contiguity of the spheres and this order imply that the maximum distance of the moon is equal to the minimum distance of Mercury (let us call it D), the maximum distance of Mercury is equal to the minimum distance of Venus (D), and the maximum distance of Venus is equal to the minimum distance of the sun (D).

30 Although there is no indication of the degree of certainty he attached to the models he developed *in Almagesia*, and although the fact that he modified some elements, such as his theory of latitudes, would suggest that he did not consider them to be definitive, the caution of his remarks in Almagesia suggests the same. The caution of his remarks in *the Almagest* suggests the same.

31 Ptolemy, *Almagest,* V, 13 for the distances of the moon to the syzygies and quadratures, and V, 15 for that of the sun. Ptolemy calculates the average distances of the luminaries, which he expresses in terrestrial radii: 59 for the moon at ywBi=, and J 2 J 0 for the sun. His models make it easy to deduce the minimum and maximum distances.

32 See chapter 3, part three.

33 See Chapter 3, Part 1.

Venus orb

CPCt

Fig. 5.5

Il already knows D and D.." All that remains is to obtain D;. But he can do much more with his hypothesis of contipuit é and the relative distances between Venus and Mercury. D is sufficient to obtain D›, and he can then calculate D based on these same hypotheses and compare this result with that obtained in / 'd/oi‹iye.›ie

Thus

D,, Dt= 88/? 4

D,= 88/34 x D,

$D_3 / D_2 = 104/16$

$\Leftrightarrow$ $D_3 = 104/16 \times D_2$

D= l O4/ 16 x B8/34 x D

The value that Ptolemy thus obtains for D is certainly different from that which he obtained for the minimum distance of the sun in */'A/fli‹iyr•.sfr.,* but it is sufficiently close to it that he does not question his assumptions. 11 is

34 Calculés dans *l'Almageste*, V, 13 & 15.

thus conformed to the opinion that Venus and Mercury are well situated under the sun." He does not attempt to account for the difference precisely, simply mentioning that it may be due to an error in the calculation of D. He also calculates the minimum and maximum distances of the superior planets and the sphere of the fixed stars, based on the ratio of these distances for each celestial body and on the maximum distance from the Sun. He thus obtains a sphere of fixed stars located at approximately 5.694633 x 10' stadia, or approximately 8.969 x 10' km (one stadium being equal to approximately 157.5 m).

This certainly seems like a sure way to achieve this end, except that, as Toomer points out, what he obtains is no longer consistent with the phenomena: according to the distances he calculates for Mercury, the latter should exhibit a significant parallax·

I I e iti n éiste ol ue de Theon et Poé e

At the beginning of this chapter, we referred to Pierre Duhem's long-influential thesis that Greek astronomy was divided into two schools, namely the proponents of a purely mathematical astronomy and those of an astronomy that took physical requirements into account. Ptolemy "was considered

35 He certainly is not in the idea that the spheres are contiguous, especially since the falsity of this hypothesis could explain the difference he obtains, to which he does not, however, allude.
Assuming Venus is below Mercury would imply the following: D2fD1= 104/16
m D2= 104/16 x D1

D3/D2= 88/34
<r D3= 88/34 xD2

D3= 88/34 x104/16 x D1
= 104/16 x 88/34 x D1, therefore the same value as for Mercury, which is less than Venus.

36 For comparison, the distance to the sphere of fixed stars that Archimedes calculated in *Arena* on the basis of Aristarchus' cosmos was 5" stadia, or approximately 7.88x 10'° km, i.e. nearly 100,000 times greater!
37 G.J. Toomer, *Ptolemy's Almagest,* p.420, note 4. However, Ptolemy had explicitly stated that this model should not lead to an observable parallax for Mercury and Venus. (Ptolemy, */'Almagesle,* IX.1, translation by G.J. Toomer, p.420).

belonging to the former, and Duhem attributed to him a "positivist" approach to astronomy, which he contrasted with Theon's "realism." Although he did not emphasize this aspect, it is clear that Duhem's thesis concerning Ptolemy was based on the fact that Ptolemy refers to his views on the physical nature of the sky only extremely rarely in *the Almagest:* only a few paragraphs address the question and, apart from these, Ptolemy deals with the movements of the stars solely in terms of geometric abstractions. As we saw in the first part of this chapter, Goldstein's discovery clearly showed that this silence cannot be attributed to a disregard for physical astronomy and called into question the notion of a positivist attitude on Ptolemy's part.

Nevertheless, this does not imply that the opposition between mathematical astronomy and physical astronomy had become meaningless at the time we are concerned with. The scarcity of references to the physical aspect of the heavens in *f'A/magrsfe* remains striking, and while this may be attributed to a tendency toward specialization on the part of Ptolemy, it is undoubtedly significant that Theon shares this same tendency. The latter does, of course, deal with both mathematical astronomy and physical astronomy in his work. However, he separates the two aspects, and, like Ptolemy, who composed *the Almagest* before the *Book of Hypotheses,* Theon first presents the mathematical models before delivering the physical models. This is all the more remarkable since he has far less reason to do so than Ptolemy, since his treatise is in a way what we would today call a popular work: one might therefore expect less specialization in his approach to astronomy than in Ptolemy, who, after all, quite naturally focused on the aspect addressed by his research when presenting its results—although he did so in a surprisingly systematic manner.

38 This is confirmed by Theon's references to the 'qoTsporixoi, whose identity he does not explicitly specify; however, they do not appear to be contemporaries. Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.233.

Furthermore, regardless of their other opinions on the physical appearance of the heavens, both astronomers insist surprisingly heavily on the fact that, mathematically, several hypotheses are possible because they are equivalent. As Duhem notes, there had been agreement on this point for a long time. The insistence with which both Theon and Ptolemy emphasize this mathematical equivalence is all the more remarkable. Ptolemy states:

> . for a body which displays a single invariant anomaly, a single one of the above hypotheses will suffice; and [in this case] all the phenomena will be represented, with no difference, by either hypothesis, provided that the same ratios are preserved in both. [...] If these conditions are fulfilled, the identical phenomena will result from either hypothesis.

Admittedly, Ptolemy mentions his preference for the eccentric hypothesis over the epicyclic hypothesis in the context of solar theory, but this preference is mainly due to formal reasons: it seems simpler to him because it involves only one movement instead of two. Furthermore, throughout his study of the movements of the sun, he offers two demonstrations for each question, one in terms of the eccentric hypothesis and the other in terms of the epicyclic hypothesis. This remains true for the models of the moon and the planets. Whenever he expresses a preference for one of the two hypotheses, his choice is motivated by considerations of mathematical practicality. For example, in the case of the moon, he chooses to represent the second anomaly by the eccentric hypothesis for reasons of ease (he uses a deferent whose center rotates around the earth), and it is only as a consequence of this choice that he represents the first anomaly in terms of the epicyclic hypothesis, in order to be able to combine them more easily. And above all, even though he expresses such preferences, he takes care to emphasize that "in fact, both hypotheses can be used."

Theon, for his part, reports with satisfaction that Adrastus has shown that both hypotheses account for the phenomena, and, still referring to these same hypotheses,

1 Ptolemy, ***Almagest,*** translated by G. J. Toomer, pp. 144–145.
2 Ptolemy, *Almagest,* translation by G. J. Toomer, p. t8l.

that Hipparchus insisted on the necessity of explaining every phenomenon by as many hypotheses as possible. He demonstrates separately each of the meanings of equivalence: first, following Adrastus, he shows that the hypothesis of the eccentric is "a consequence of that of the epicycle" that is, that a celestial body moving on an epicycle also describes an eccentric circle. He even attempts to provide two different demonstrations of this—although in fact the first is not valid for the entire orbit but only for four positions: the apogee, the perigee, and the *mediae longitudines.* He then demonstrates the reverse: "...to tell the truth, the hypothesis of the epicycle is also a consequence of that of the eccentric."

Furthermore, as we saw in Chapter 2, in terms of mathematical astronomy, the way in which Ptolemy demonstrates the equivalence of the two hypotheses is as far removed as possible from physical considerations, and would certainly support a positivist approach. While Theon considers only the case where the eccentricity and the deferent have the same radius and where the eccentricity is equal to the raypn of the epicycle (Fig. IJ (he demonstrates that in these circumstances, regardless of the hypothesis chosen, the sun will also describe the path defined by the other hypothesis),' Ptolemy considers a much more general case: the eccentric circle and the deferent can be of different sizes and the eccentricity may not be equal to the radius of the epicycle, provided that the ratio of the eccentricity to the radius of the eccentric circle in the eccentric hypothesis is equal to the ratio of the radius of the epicycle to the radius of the deferent in the epicyclic hypothesis (Fig. 2). He demonstrates that, as seen from Earth, the sun will then appear in the same direction. This truly expresses the equivalence of the two hypotheses in terms of appearances, since the body is generally located in different places in the sky. And

3 Theon of Smyrna, Exposition, translated by J. Dupuis, p.269, Adraste shows how the phenomena are explained in both hypotheses, that of the eccentric and that of the epicycle. Hipparchus pointed out that it is worthy of the mathematician's attention to seek an explanation for the same phenomena using such different hypotheses, that of eccentric circles and that of concentric circles and epicycles.

4 Theon of Smyrna, *Exposition,* translated by J. Dupuis, p. 269, "Adrastus showed that the eccentric hypothesis is a consequence of the epicycle hypothesis."

5 See ch. 2, p. 71.

6 Theon of Smyrna, *Exposition,* translation by J. Dupuis, p.269.

7 In both figures, the Earth is represented by B.

8 Ptolemy first presents a similar demonstration—although even at this stage he tends to insist that, consequently, the same thing will be seen from Earth, which Theon does not do.

It is indeed this equivalence that Ptolemy alludes to in the excerpt quoted above, as indicated by his reference to "the same ratios [being] preserved in both hypotheses."

Theon:

Ptolemy:

0

0

Fig.S .6a

Fig. 5.6b

Finally, although Theon and Ptolemy both practiced both mathematical astronomy and physical astronomy, there is nevertheless a significant difference in the relative importance they attached to them. Indeed, when introducing the subject of ***the Almagest,*** Ptolemy expresses the opinion that astronomy is the highest intellectual activity, he is referring to mathematical astronomy. It is clear that what he values above all else is stable knowledge and

9 This is not the only case in which Ptolemy disregards fundamental considerations. The same applies to his model of the moon, which is a major shortcoming of his model (Olaf Pedersen, A ***Survey of the*** *Almagest,* pp.198-199 & p.207): the size of **the epicycle** leads to differences in the Earth-Moon distance such that the apparent diameter of the moon at perigee (quadratures) should be approximately twice what it is at apogee (**syzygies).** In this case, the lack of attention to radial considerations does not even allow us to "save appearances" in the positivist sense, because the model does not even conform to this criterion.

reliable, even certainty; he seeks to achieve something unchanging, something that, once obtained, will be unlikely to need revision." This is an almost spiritual quest: a way of approaching the divine, whose essence is eternity and permanence. However, of the three branches of "theoretical philosophy" that he considers—in which he follows Aristotle—only mathematics seems reliable. Indeed, he states with reference to physics and theology:

> ... the first two divisions of theoretical philosophy (theology and physics) should rather be called guesswork than knowledge, theology because of its completely invisible and incomprehensible nature, physics because of the unstable and unclear nature of matter; hence there is no hope that philosophers will ever agree about them; and [...] only mathematics can provide sure and unshakeable knowledge to its devotees, provided one approaches it rigorously. For its kind of proof proceeds by indisputable methods, namely arithmetic and geometry.

It is therefore as a branch of mathematics that he considers astronomy to be the highest form of knowledge. His preference for it over other forms of mathematics is motivated by the same considerations. He continues:

> Hence we were drawn to the investigation of that part of theoretical philosophy [mathematics], as far as possible to the whole of it, but especially to the theory concerning divine and heavenly things. For that alone is devoted to the investigation of the eternally unchanging."

In this respect, Ptolemy's perception of mathematics and astronomy is very different from that of Theon. In general, Theon's attitude towards mathematics is much more ambiguous. Although some aspects of his work are clearly

10 This does not mean that they were dogmas. On the contrary, he implicitly mentions the possibility that observations made over a longer period of time than that available to him could lead to different views on such fundamental concepts as the immobility of the stars relative to each other. (Ptolemy, *Almagest,* translation by G. J. Toomer, p .321).
11 Ptolemy, *Almagest,* translation by G. J. Toomer, p.36.
12 Ptolemy, *Almagest,* translation by G. J. Toomer, p.36.

Pythagoreans and qu[s] he considered mathematics to be an important means of developing the mind, for him it was merely a prerequisite for what he considered to be the true objects of intellectual activity — a little like we consider literacy. He expresses this clearly in the introduction to his treatise:

> ...Plato himself does not want us to continue into extreme old age drawing geometric figures or singing songs, things that are suitable for children and that are intended to prepare and purify their minds, to enable them to understand philosophy. [3]

These views certainly influenced Théc's approach to astronomy. He explicitly states his preference for a physical rather than a strictly mathematical approach to this discipline, although, once again, his attitude towards the former is not without contempt:

> *All this leads us far astray under the pretext of reconciling the hypotheses and reasoning of mathematicians.* The latter, considering only the phenomena and planetary movements produced according to the course of nature, after observing them for a long time in favorable locations, in Babylonia, Chaldea, and Egypt, eagerly sought principles and hypotheses that explained the phenomena. They thus managed to confirm the facts they observed and predict future phenomena, the Chaldeans using arithmetic methods, the Egyptians using graphical methods, *all using imperfect methods and without sufficient knowledge of nature, 'for the joys must also be discussed from a* physical *point of view*.'(4)

As we have seen, at the end of this process, Theon arrives at a single model, no doubt because imagining a physical model that would be the materialization of the eccentric would be too complex. This leads him to conclude that considering the physical aspect of the heavens allows us to

13 **Theon** of Smyrna, ***Exposition,*** introduction by **1. Dupuis,** p.2S.
14 Theon of Smyrna, *Exposition,* translation by **1. Dupuis,** p.2R7, my notes.

determine their actual arrangement, whereas mathematical astronomy is insufficient for this task:

> ·of the two hypotheses ... that of the epicycle seems [...] most consistent with the nature of things. For the epicycle is a large circle of the solid sphere, the one that the planet describes in its motion on this sphere, while the eccentric differs entirely from the circle that is consistent with nature, and is rather described by accident,'[5]

And again:

> ...it [the movement] follows the epicycle when it appears to follow the eccentric. It is obviously reasonable that there should be agreement between the two hypotheses of mathematicians on the movements of the stars, that of the epicycle and that of the eccentric; both happen to agree with the one that is consistent with the nature of things..."

Given the importance he attached to mathematics and the trust he placed in it, it is doubtful that Ptolemy considered physical astronomy capable of resolving questions concerning the motion of the stars that could not be solved by a purely geometric approach. It seems fairly clear that he intended first to determine geometric models and then to deduce the physical reality of the heavens by "materializing" these models. This is certainly suggested by the general organization of his work, which consisted first of ***the Almagest and*** then the Book of Hypotheses. He may well have changed his mind while writing ***the Almagest,*** especially since he felt compelled to justify the 'complexity' of his results towards the end of the work. However, the somewhat unsatisfactory values he obtained when attempting to calculate the distances of the orbs based on physical considerations in the *Book of Hypotheses* did not prompt him to

IS Theon of Smyrna, *Mposition,* translation by 4. Dupuis, pp. 303-305.
16 Tliéon of Smyrna, *Erposi/ioii,* translation by J. Dupuîs, p.299.
17 Certes Théon also addresses the questions in this order, but this choice cannot be considered as significant as it is for Ptolemy, for whom it is part of his research program itself.

questioning the ratios of the distances he had calculated for each star, suggesting that he placed greater trust in these calculations.

We can therefore assume that, unlike Theon, Ptolemy always placed greater trust in a geometric rather than a physical approach to astronomy. Nevertheless, the difference between the two astronomers was one of method, not of objective. The quest for the absolute that motivated Ptolemy's interest in mathematical astronomy itself indicates that he was deeply concerned with the reality of the heavens and their divine inhabitants.

Furthermore, his statements at the beginning of *the Almagest,* to the effect that 'physics' cannot be a source of reliable knowledge, in no way contradict his interest in physical astronomy: what he is referring to are terrestrial processes, in accordance with Aristotelian thinking, since he alludes to 'the unstable and unclear nature of matter'. The argument therefore has nothing to do with possible difficulties inherent in any conceptual representation of reality, and is clearly inappropriate when it comes to the heavens. In fact, the fact that the latter do not share this 'unstable' character is precisely one of the ideas that motivated his interest in astronomy; and this eternal character, like Aristotle, he attributes to the ether which, despite the subtlety of its qualities, Ptolemy conceives as entirely 'physical' in the modern sense of the term.

In fact, in the very passage that Duhem quotes to support the idea of a positivist attitude on the part of Ptolemy, it seems that in reality he is referring to spheres and axes. Because of its importance and subtlety, this passage deserves to be quoted at length and without interruption. The corresponding passages in Duhem are indicated in the notes, along with the order in which he presents them. The excerpt from this passage that Duhem does not quote is indicated in bold in the translation below, which we owe to Toomer:

18 As Olaf Pedersen points out, *A Siirvej' of the Almagest,* p.394.
19 Talking about the physical nature of the heavens would therefore be contradictory given his conception of the term 'physical'.

Now let us consider the complicated nature of our devices and judge such hypotheses to be over-elaborated. For it is not appropriate to compare human [constructions] with divine ones, nor to form one's beliefs about such great things on the basis of very dissimilar analogies. For what could one compare more dissimilar than the eternal and unchanging with the ever-changing, or that which can be hindered by anything with that which cannot be hindered even by itself?[0] Rather, one should try, as far as possible, to fit the simpler hypotheses to the heavenly motions, but if this does not succeed, [one should apply hypotheses] which do fit. [2] 'For provided that each of the plenomerta is duly saved by the hypothesis, why should anyone think that such explanations should not characterize the motions of the heavens when their nature is such as to allow no interference, but of a kind to yield and give way to the natural motions of each part, even if [the motions] are opposed to one another? Thus, quite simply, all the elements can easily pass through and be seen through all other elements, and this ease of transit applies not only to the individual circles, but to the spheres themselves and the axes of revolution. We see that in the models constructed on earth, the fitting of these elements to represent the destructive motions i s very good, and difficult to achieve in such a way that the motions do not
hinder *each* other, while in time no obstruction whatsoever is caused by such collisions. Rather, we should not judge 'simplicity' in heavenly things from what appears to be simple on earth, ...*

We are thus faced with a problem, perhaps one of translation. Indeed, Duhem, who

uses Halma's translation, does not mention what is undoubtedly the most important passage.

20 "Quc I on ailm pas constděrw Jm constructions **quc** nous a›"ons **a6cnc** es,afin dc **jugcr par** II les diffžcufičs měmc des h›potl čscs. *It* is not appropriate to compare human things with un**divines: il** things. We should not base our confidence in objects that are so different by relying on examples that differ greatly from them. Is there anything in cffcL that differs more from the immutable laws than *the* constantly **changing** *letters*? The letters that are subject to the contmintc ¢lc I"Uni\'criticize the alternative **characters** that are not subject to the constraint that they express. Rerrs Duhem. Czu/azv ro yxz ›'oyc o, p. i8. Third passage quoted by Dubem.

21 "fl faug du **nieux** qu"on 1c §cut, uda[zer **les I»pxhčs9s**)es plus siniplm aiLx inou\e›ncnts cčlcstcs ; Iriais si cllcs nc suJTisent il faut cn prendre d'autzcs qui cmn›'ienncat nlieux.", Piezze Duhern,€ civ co y›zz7p'o,ucr'o. p, **t7. First** passage quoted by Duhcm.

22 "In clf I. ri cliacun dcs nioti›'cn1oltts app rcnts sc troti›c sau\'č d lilrc dc cvnsdqficncč dcs h pothčscs. who therefore gourrnil Iroux cr tonn4nt quc. dc cos njo tx'cnicnts comptic}\tčs, rčsut\eni the nlottx'cnzc»ts dcs corps cčlcstcs'/" Rerre Duhem. SzvJc/v za y›azvo//crzr, pp.17-18. Second passage quoted by Duhem.

21 *fl n s' a, dgMs la rčgion oči sc produiscnt ccs iuous' jenl aucunc ms¢ncc qcii soiL pnr najurc, ¢louča dc **I:i puisancc** @ opposes * ccs **mmn'cmvuts; sc which s** 1' trowyc cčdg ax"cc **indifičrcncc** aux mou "cn\cnts n.aturcls dc chacun dcs astt‹s ci les laissc passcr. bicn quc ees tnous eincnts sc produiscnt cn dcs scns opposčs; m soric quc tous les astm pcw'cnt passcr. ct quc tous pcuvcnt čtrc agcryw. au lm›'crr dc tous lee fluides qtn sont téjxindus d' unc nianiùm lsomogénc." Pierre Ouhem. £mÇcir ra çxzi ro,ucrs. pp.18-19. Fifth passage quoted by Duhem.

24 "Tani que- rioiis consider these constitutional representations for us, we find ourselves in agreement with the composition of the succession of the new rulers: the at,enccr in such a way that each of them can be carried out freely"nicnÎ seems to us an impracticable task. But if we examine this rJiii, what happens? Then Heaven. we are no longer bound by a scandalous mixture of new ideas." Pierre Ouhem. jam ra pur room, p.18. Fourth passage quoted by Duhem.

2-S /, .4fiiinpesr#. Translation by G.J.Toomer. pp.60tï-6tll.

more fundamental to the subject at hand, namely the reference to "the spheres themselves and the axes of revolution" that appears in G.J. Toomer's translation. It is therefore the translation available to us here that suggests an interpretation of this passage that is not only different, but even completely opposite to that of Duhem.

As the latter notes, Ptolemy undertakes here to respond to possible objections from his readers—especially, one might assume, contemporary astronomers—who might criticize him for the "complexity" of his models. In this context, what Ptolemy is referring to with this term is nothing less than a physical impossibility: the various components involved would interfere with each other. But contrary to Duhem's interpretation, Ptolemy's response, according to Toomer's translation, just as realistic as the challenge he expects: he seems to justify the validity of his system by invoking the physical properties of the ether, which allow the spheres and axes to exist as such while being able to pass through each other—and this without effort, without disturbing the regularity of the heavens by slowing each other down.

It should also be noted that, contrary to what this type of argument suggests, Ptolemy does not hesitate to recommend in the Book *of Hypotheses* that an attempt be made to construct a material model of the Earth to represent the movements of the stars, for educational purposes. This again contradicts Duhem's interpretation, which contrasts models such as those of Theon, which "can be represented by the art of the turner," with those of Rolémée.

It would therefore seem that, although different astronomers placed greater importance and trust in one or the other approach to astronomy—i.e., physical or mathematical—this probably did not imply an epistemological difference on their part, at least up to and including Ptolemy. This is hardly surprising given what is undoubtedly the most fundamental postulate of Hellenistic astronomy: the movements of celestial bodies appear irregular when viewed from Earth, whereas *in fact* they must be regular. It is difficult to imagine what

26 Ptolemy, *Book of Hypotheses*, Treatise 2, translation by Régis Morelon (notes), p.7.

ariire sipnification if it is not realistic, we can give these circular innovations meaning based on the fact that this idea makes sense. It therefore seems clear that for these astronomers, phenomena or appearances do not represent the limit beyond which the human mind cannot hope to penetrate. Therefore, the closed circle that we are concerned with is not so much the phenomena themselves as the way in which they are perceived. they constitute the deceptive surface of a very different reality which, due to its clivine character, cannot but matter—as in the case of Ptolemy II's quest for the absolute, it would seem that astronomy consisted as much in knowing the days of appearances as in knowing the latter themselves.

CONCLUSIONS

Comparing the works of Theon and Rolémée as we have done here sheds light on the scientific context in which these two astronomers worked, and thus allows us to better understand Rolémée's contributions. Some of the conclusions that can be drawn from this study tend to confirm what Rolémée's comments in ***the Almagest*** suggested about the work of his predecessors. Thus, with regard to models for movements in longitude, it appears on the one hand that the equant was most certainly absent from those that existed in Theon's time, as he makes no mention of it. It therefore seems clear that it was Rolémée himself who took responsibility for introducing this element, which implies a violation of the principles laid down by Plato. In , although he does not specify this explicitly, it is clear that the motion to which he refers must be circular and uniform *with respect to a **single point,*** which is what the equant puts an end to, since the motion appears uniform when viewed from its center, while it always proceeds in a circular manner along the deferent.

On the other hand, it seems that at the time of **Theon,** models did not include an eccentric deferent. It was therefore Rolémée who merged the two hypotheses: previously, two possible ways of accounting for the movement of each celestial body, corresponding to slightly more complex versions of the epicycle and eccentric hypotheses. According to Theon's treatise, these two models were designed according to the same standard scheme, to the extent that they can easily be considered to be the same models for all planets, with only their parameters varying. A major contribution of Rolémée would therefore be to have arrived at a unique solution for each wandering star (since the movement of each of them can no longer be represented by two different hypotheses), and to a diversity hitherto unknown among the models of different celestial bodies. In this sense, when considering the model of a given planet, we can say that what Ptolemy proposed is more *comprehensive.*

It is very likely that this aspect of his results bothered him. At least, that is what we can infer from the evolution of his efforts to account for the movements of the stars in latitude. Indeed, at first (i.e., in *the Almagest),* the models proposed by Rolémée seem to have been considerably more complex than previous models. It is true that, as far as Theon's account allows us to judge, before Rolémée's work, these had hardly evolved since Hipparchus, and consisted simply of an inclination relative to the ecliptic of the plane in which the movements in longitude take place. In *the Almagest,* Rhoemeteus made this inclination variable and added, for certain planets, two back-and-forth movements of the epicycle around two axes, one parallel to the inclined deferent, the other passing through the Earth and the center of the epicycle. But later, in the *Book of Hypotheses, he* simplified these models by eliminating all variations in the inclinations of the deferents and postulating a constant inclination of the epicycle for the inferior planets. Furthermore, these models differ more from each other in *the Almagest* than in *the Book of Hypotheses.* This is of course linked to the simplification that

'l rois ä part jx›or le soleil.

We have just mentioned this, but it would also suggest that Ptolemy preferred greater uniqueness in his models, and that it was truly against his will, compelled by the need to account for phenomena to which he attached even greater importance, that he had to abandon it.

In addition to Ptolemy's contributions to the models of individual stars, Theon's work reveals a particularly original hypothesis when compared to what was developed later— based on the models of Rhoemete, of course. Theon considers the possibility that Venus and Mercury revolve around the sun.
— and with the latter around the earth. 11 It is nevertheless difficult to estimate how popular this model was in the time of Theon and Rolémée. The fact that Theon mentions it would suggest that it was commonly accepted as possible, but Rolémée's complete silence on the subject suggests that he was not familiar with it.

There are also differences in the reasoning styles of the two astronomers. In particular, Theon tends to want to demonstrate that the scenarios he presents constitute all possible hypotheses, and tries to list them all. Ptolemy, on the other hand, often refrains from presenting alternatives and assumes that the success of his theories will sufficiently prove the validity of the hypotheses on which they are based. They give us several examples of this difference in attitude, which cannot be explained entirely by the fact that they were addressing very different audiences. Thus, when he sets out the hypotheses of the eccentric and the epicycle, Theon attempts to show that these are the only possible ones, since they correspond to positions of the solar orb that would either surround the earth or be outside it. He even takes the trouble to explain that the third case, that of a position of the earth within the solar orb, is untenable. Rolémée, for his part, does not mention this third alternative and gives the impression that there could well be other equally valid hypotheses. He does not seek to exhaust all possibilities. We can therefore say that, when it comes to listing hypotheses, Ptolemy's approach is less 'demonstrative' than Theon's.

Similarly, when considering the epicyclic hypothesis to explain the movements of the sun, Theon considers all possible cases for the directions of rotation of the epicycle on the deferent and of the star on the epicycle. Ptolemy, on the other hand, states his choice from the outset and sets out to show that it accounts for the movement of the sun, barely mentioning the alternatives.

These particularities can be found in the way the two astronomers approach the problem of the shape of the Earth. Rolémée considers shapes other than spherical and shows that they are inconsistent with the phenomena. Theon, on the other hand, sets out to demonstrate that the Earth is spherical by first proving that it is connected in the east-west direction, then in the north-south direction. Theon's approach is therefore once again more demonstrative, while Ptolemy tends to proceed by refuting competing hypotheses. However, the way in which Ptolemy expresses his interest in astronomy shows that he is motivated by a need for absolutes, for certainty. His demonstrative approach is therefore quite different from what his convictions might have suggested.

Finally, comparing the approaches of Theon and Ptolemy is also of great interest from an epistemological point of view, as Duhem had already pointed out. Several elements, including, above all, the re-evaluation of Ptolemy's attitude to which Goldstein's discovery invites us, seriously call into question Duhem's conclusions in this regard. It appears that. if Ptolemy did indeed accord greater importance and greater credit to mathematical astronomy than to physical astronomy, this does not imply a 'positivist' attitude on his part. On the contrary, the complete version of *Liyre* cfes Nyyof/ieses shows that he was sufficiently concerned with the reality of the heavens to develop physical models of them, attempting to materialize the geometric models developed in *the Almagest.* Only the equant, whose sole interest lies in the position of its center, did not find materialization, as the stars had to respect the uniformity of their movements in relation to this center of their own accord. It is of course possible that after Ptolemy, Proclus developed a positivist perception of astronomy, as Duhem asserts. But this is not true of the Alexandrian astronomer Ptolemy, who shared Theon's realistic conception of astronomy, and they disagreed only on the question of which of the two approaches, physical or mathematical, was the best means of accessing knowledge of this celestial reality.

2 Pierre Duhcm. *P‹ojsi ' ru iyai vopcva,* p.2t).

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